

## Conclusion

In this study, students had the lower order skills of rote memorization. However, they were not able to relate or apply information from previous instruction to the concepts. This may be due to 1) years of passive education or 2) memorization of fragmented facts. Or they might not have been successfully taught to think critically or realize when they are not doing so.

We must teach students to reason with discipline, skill, and flexibility in order to become critical observers and communicators, and be prepared for the technological society we are producing. Students must learn these skills to be functional in an age of accelerating change. Without these skills, students are destined to become merely users and servants of technology, vulnerable to the lowest forms of persuasion, and incapable of reasoning through information, misinformation and disinformation put forth by the media. Indeed, the functional distance between the few educated elite and the numerous "trained" mediocre may become a crippling societal problem.

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**Arleen J. Watkins** is currently a Research Specialist for Cancer Research and the Coordinator of Third Year Medical Students at the University of Arizona. She also coordinated the self study program in introductory mathematics at the University of Arizona.

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## Research & Teaching in Developmental Education

By Arthur B. Powell, Esaié Pierre and Carlos Ramos, Rutgers University

### **Researching, Reading, and Writing about Writing to Learn Mathematics: Pedagogy and Product**

#### Abstract

The collaborative work of a mathematics professor and two undergraduate students to produce an annotated bibliography about writing to learn mathematics offers a new research paradigm. The authors discuss this paradigm, procedural and affective aspects of compiling and annotating the bibliography, and criteria for selecting bibliographic entries. At the end of the article, the authors present their annotated bibliography.

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Researching, reading and writing are activities usually associated with intellectual or academic projects. Though the project we describe entailed these activities, nevertheless, it was unusual for two essential reasons. First, it involved a teacher and, rather than doctoral students, two undergraduate students, together laboring to annotate a bibliography of selected articles and books about writing to learn mathematics. This tangible product was one focus of the project. As important and as useful as bibliographies are, more significant was the second focus. This was pedagogical and was connected to developmental education, specifically to mathematics. In both anticipated and unanticipated ways, the project proved particularly powerful in encouraging students -- who had been underprepared mathematically -- to participate in mathematics and to contribute to mathematics education. Moreover, the pedagogical and collaborative processes of this project, including its product orientation, signal the emergence of a new paradigm for research activities in developmental education (Powell, Jeffries, & Selby, 1989). In this paradigm, students and teachers collaboratively investigate and develop curricular materials as well as pedagogical techniques.<sup>1</sup>

The two students collaborating on this project, Pierre and Ramos, had taken a "prealgebra" course that Powell taught and, in it, had to maintain journals (or learning

logs).<sup>2</sup> They performed well in the course and, in addition, both orally and in their journals, raised challenging questions about the purpose and appropriateness of writing in that and, indeed, in any mathematics course. This intrigued Powell, who after the course ended invited them to participate in his bibliographic project, which lasted for two years. During the project, independently, each decided to pursue mathematics beyond what the College strictly requires for a baccalaureate degree.<sup>3</sup> One added mathematics as a minor while the other changed his major from history to mathematics. Though their increased involvement with mathematics occurred after they began the project, we are not claiming causality. Nevertheless, the pedagogical focus of the project involved Pierre and Ramos in researching, reading, and writing about uses of writing for learning mathematics. Through the project, Powell encouraged them to continue to use writing in their study of higher mathematics. In the article, describing their roles in the project, the authors discuss whether this writing continued and whether Pierre and Ramos derived other benefits. Then the authors elaborate upon the theoretical and pedagogical criteria for selecting articles and books for the annotated bibliography.

### **Project Activities and Benefits**

When the project began, Pierre and Ramos had different conceptions of the project and feelings about participating. Ramos thought that Powell's invitation offered him an interesting work-study assignment. In other words, he viewed the project as merely a job, with little else tied to it. Pierre, on the other hand, was surprised when Powell invited him to participate, he did not believe himself to be a good enough reader or writer. However, he trusted that Powell believed him to be capable, and thus joined the project with reticent enthusiasm. Moreover, Pierre having used writing in a mathematics class, held preconceived notions about what the authors would argue.

Pierre and Ramos not only had different initial attitudes toward and conceptions of the project but they had different experiences with researching, reading, and writing. While both performed the same duties—researching the literature, reading articles, and writing abstracts—they, nevertheless, developed differently. For example, discussing with Powell each article and abstract ameliorated Pierre's confidence about his ability to read and write. Furthermore, since these discussions centered on evaluating and identifying important points in the articles, Pierre began to perceive salient points about the substance of subsequent articles and to incorporate these points into the abstracts he wrote. Thus, as he grew more confident about his reading, he developed his facility to write abstracts.

Ramos also developed further his ability to read critically. To capture the meaning of each sentence, he read the articles slowly and deliberately to remain in touch with an author's thoughts as he moved from sentence to sentence and paragraph to paragraph. He also extended this approach to writing abstracts. Initially, he would write all that he wanted and thought was important to say and then would slowly and deliberately recast his commentary.

Along with developing careful and critical reading and writing strategies, we observed other benefits and outcomes. Since Pierre had used writing in a

prior mathematics course, he believed that he knew its limitations and benefits. From the readings, he discovered benefits that went beyond his initial conceptions. He learned that differences in the needs of individuals meant that they derived different benefits from using writing as a learning tool. Also, he changed his original conception that writing was a learning tool confined to mathematics.

Learning about theories and practices of using writing as a learning tool led both Pierre and Ramos to adopt new study strategies and to appreciate how writing facilitates learning. Ramos, for instance, felt able to discuss the articles he read with the authors. Also, he learned about "reader expectation theory"<sup>4</sup> and applied it whenever he wrote. Pierre incorporated writing in studying precalculus, calculus, genetics, and chemistry whereas Ramos used writing to study linear algebra and differential equations.

Overall, Pierre and Ramos derived a number of benefits from this project. Perhaps the most salient is that they came to understand and appreciate more deeply the usefulness of writing for learning. Similar to the effects of reading and writing about it, the process of writing to learn is like the morning dew that forms on windows. At the start of the night, the window is dry, comparable to the state when understanding a subject is nil. By the appearance of light, moisture condenses on the pane, and ultimately water droplets stream down the glass. In the same way, writing to learn develops and solidifies thoughts and ideas in the minds of students. The accumulation of moisture on the pane is comparable to the accumulation of insights that students acquire as they write to learn. Each time they write about a piece of mathematics, they must reflect on it and accommodate their thoughts and ideas to words. These processes force students to become aware of the contents of their minds and to think more clearly, consequently, they solidify their understandings of mathematics and generate mathematical knowledge.

### **Writing to Learn Mathematics and Selection Criteria**

The above metaphor illustrates the justification proffered in the literature for the efficacy of writing as a learning tool. From a corpus on writing to learn mathematics, published in English, mainly in North America, we used specific criteria to select references for the bibliography. To provide a context for these criteria, we first briefly offer related historical and theoretical issues.

Since about 1977 and particularly after 1987, discussions have proliferated in the literature and at professional gatherings concerning the efficacy of writing as an adjunct to the teaching and learning of mathematics.<sup>5</sup> Among mathematics educators, this interest appeared in the wake of, as Hairston (1982) describes it, a Kuhnian revolution in theoretical and related pedagogical theories of composition. Insurgent perspectives in cognitive psychology, neuropsychology, sociolinguistics and philosophy informed new pedagogies through which educators viewed writing not only as a communicative tool but also as a powerful instrument for reflecting on experiences. Educators began to understand that, like mathematics, writing is a major tool for thought, and that, most importantly, students could use it as a tool for learning disciplines other than English and literary criticism. Indeed, even before this revolution in the teaching of writing began, Bruner (1968) advised that both writing

and mathematics are instruments or "devices for ordering thoughts about things and thoughts about thoughts" (p. 112).<sup>6</sup> Reasonably, therefore, one can imagine mathematics educators exploring pedagogical tools whereby these seemingly dissimilar instruments function, as it were, symbiotically to augment learning.

From our investigation of nearly one hundred works, we discovered that mathematics educators in increasing numbers are exploring ways of employing writing as a support for mathematics learning. Naturally, among these explorations, there are significant differences in the rationales and purposes for and implementations of writing into mathematics curricula. Nevertheless, among these differing approaches, two broad categories are identified: product approaches and process-product approaches. In the former approach, writing is used as a way of *demonstrating* knowledge, in process-product approaches, writing is used as a way of *achieving* knowledge or, in other words, as a way of *knowing*. In product approaches, mathematics educators engage students in writing activities for purposes that immediately focus on mathematics rather than on students. The concern is for what students know at the moment, not for how their mathematical ideas evolve. In contrast, when educators design writing activities in the process-product category, they intend to stimulate and support students' evolving understandings and insights, to encourage students to express their affective responses to mathematics; and to prompt students to write and analyze their mathematical autobiographies or histories. In this category, writing activities focus on the intellectual and emotional energies of students by prompting them to reflect broadly and deeply on mathematics, on their learning, and on their feelings. Consequently, writing activities in this category tend to have several phases or drafting stages. Moreover, through these stages, students have opportunities to negotiate meaning and generate knowledge.

Pedagogical differences suggested by the two categories of approaches require student writings to have different functions. These functions range between expressive and transactional writing, two of the three categories formulated by Britton *et al.* (1975): Transactional writing is chiefly the type of writing that students produce in product approaches. In these approaches, writing activities are used for assessment and diagnosis, for students to record step by step arithmetic procedures, and for assignments in which students, responding to instructor-supplied questions or topics, complete sentences or write short, well-developed passages. Since the primary aim of product approaches is evaluation, writing activities in these approaches require students to produce writing that is impersonal or transactional rather than expressive.

With substantially different aims from product approaches, writing activities in process-product approaches function to evince students' independent and divergent thinking. Such activities tend to require exploratory, speculative writing in which students externalize some of the content of their minds. Writing here is used primarily as a vehicle to learn about mathematics and oneself as a learner, not merely to measure the extent to which students memorize information. Furthermore, though mathematics educators do not necessarily describe their writing assignments using the theoretical categories of Britton *et al.*, many process-product activities move students dialectically and recursively along the expressive-transactional continuum.

A characteristic of expressive writing is that, according to Britton *et al.*,

"[it] is a kind of matrix from which differentiated forms of mature writing [poetic and transactional] are developed" (1975, p. 83). In process-product approaches, expressive writing activities often provide starting points for transactional writing. Students write to articulate some combination of their beliefs about the nature of mathematical knowledge, their thoughts about a piece of mathematics, and their affective responses to the mathematics they are learning. Necessarily, this involves them in negotiating meaning and generating knowledge as well as in reflecting on and monitoring their cognition and affectivity. From these expressive starting points and through processes that include response and revision, students move toward writing about mathematics so that the function of their writing is transactional.

While two broad categories of writing activities—product and process-product—are defined and the types of functions of activities that pertain to each are described, we have chosen to include only those articles and books that describe activities that fall within the process-product category. Though we recognize that writing activities that are just in the product category have their place in mathematics curricula, we believe that educators overuse them and already know very well how to devise such activities. Instead, the focus is on activities that engage students in generating and appropriating knowledge by providing opportunities for students to achieve critical distance from their first jottings or thoughts. Also omitted are empirical studies. The studies consulted were designed to measure the effectiveness of writing on student achievement in and attitudes toward mathematics. However, either the confusing and contradictory information their methods yielded or the brief instructional periods of these studies made their empirical investigations problematic.

Can students, in fact, use writing as a vehicle to learn mathematics? Many of the authors cited in the bibliography tackle this question. As well, most describe particular writing activities for accomplishing specific purposes.

### Annotated Bibliography

BeMiller, S. (1987). The mathematics workbook. In T. Fulwiler (Ed.), *The journal book* (pp. 359-366). Portsmouth, NH: Heinemann.

BeMiller argues for the term 'workbook' over 'journal' or 'log' not only to emphasize a personal approach to learning and knowledge, not only to avoid the conflicting image that some have associated with journals and mathematics, and not only to encourage constant recordings of observations but also to stress that learning results only from working. He then describes assignments he gives students involving transactional and expressive writing. Finally, he offers five soft assertions about the processes whereby writing affects learning.

Berlinghoff, W. P. (1989). Locally original mathematics through writing. In P. Connolly & T. Vilardi (Eds.), *Writing to learn mathematics and science* (pp. 88-94). New York: Teachers College Press.

Berlinghoff describes how he uses writing in his freshman mathematics course. Students are introduced to problem-solving strategies, given a problem to solve, and then asked to write about the process they used to solve the problem.

Berlinghoff claims that this form of writing refines students' mathematical thinking and that they come to appreciate this form of thinking

Birken, M. (1989) Using writing to assist learning in college mathematics classes In P. Connolly & T. Vilaridi (Eds.), *Writing to learn mathematics and science* (pp 33-47) New York: Teachers College Press

Unlike the approach of some teachers, Birken points out why writing in a mathematics class should not be a fad. She discusses benefits that students derive from writing and describes four categories of writing activities. For each activity, she provides an example of a mathematical problem that she uses in connection with it.

Borasi, R., & Rose, B. (1989) Journal writing and mathematics instruction *Educational Studies in Mathematics*, 20(4), 347-375

Borasi and Rose evaluate why writing in a mathematics course helps students. From a study involving students in a course, "Algebra for Professional Programs," they elaborate benefits that students and instructors can receive through dialogue and from journal writing.

Brandau, I. (1990) Rewriting our stories of mathematics. In A. Sterrett (Ed.), *Using writing to teach mathematics* (pp 73-77) Washington, DC: The Mathematical Association of America

Brandau invites students to write mathematical autobiographies and, through these, to re-examine their views of mathematics. Chronologically, she presents the writing of one student and documents improvement in his attitude toward both mathematics and writing. She also discusses advantages and disadvantages of mathematical autobiographies.

Buerk, D. (1982) An experience with some able women who avoid mathematics. *For the Learning of Mathematics*, 3(2), 19-24

Buerk reports on a study of math-anxious women, who were, according to the Perry scheme, generally "relativistic" but "dualistic" in their beliefs about the nature of mathematics. Before and after the study, the women were interviewed and wrote about their feelings about mathematics. They were given problems and, before solving them, asked to build mental images of the problems. Afterward, they discussed how they would solve the problems. Also, they wrote about solving the problems and about insights they derived from discussing and working on the problems.

Buerk, D. (1990) Writing in mathematics: A vehicle for development and empowerment. In A. Sterrett (Ed.), *Using writing to teach mathematics* (pp 78-84) Washington, DC: The Mathematical Association of America

Buerk describes her work in trying to change students' thinking about mathematics. She engages her students in writing and dialogue as a way to prompt them to think while doing mathematics. She describes how, using writing, two students changed their views of mathematics and problem solving.

Burton, G. M. (1985) Writing as a way of knowing in a mathematics education class. *Arithmetic Teacher*, 33, 40-45

Burton describes pedagogical uses in a number of writing assignments and their benefits for students and instructors. She suggests ways students can help each other, an example of which are guidelines for peer review. Also, she gives a format for assessing the quality of students' work.

Countryman, J. (1992). *Writing to learn mathematics: Strategies that work*, K-12. Portsmouth, NH: Heinemann

Countryman demonstrates how the use of journals, learning logs, letters, autobiographies, investigations, and formal papers can dramatically improve the reasoning abilities of students. In the book, she provides descriptions of writing activities that one can use to enhance the learning of math, and includes examples of student writing from short journal entries to excerpts from longer research papers. She also suggests topics students can explore and present at different levels of school mathematics, including descriptions of student responses to these presentations.

Driscoll, M., & Powell, A. B. (1992) Communicating in mathematics. In C. Hedley, D. Feldman, & P. Antonacci (Eds.), *Literacy across the curriculum* (pp 247-265) Norwood, NJ: Ablex

Driscoll and Powell contend that students more successfully negotiate meanings in mathematics when educators attend to communicative processes. They discuss an experiment in which writing is used to gauge students' comprehension of mathematics text. Based on the experiment, they describe a technique in which students, working collaboratively in small groups, use writing to negotiate their comprehension and interpretation of text.

Elsholz, R., & Elsholz, E. (1989) The writing process: A model for problem solving. *Journal of Mathematical Behavior*, 8, 161-166

Equating drawing as the writing activity of kindergartners, the authors describe how they adapted Donald Graves' five-step model of the writing process to the process of solving problems. They also discuss the work of one student.

Frankenstein, M. (1989) *Relearning mathematics: A different third R: Radical maths*. London: Free Association Books

In Part One of Frankenstein's book, she discusses issues in the learning of mathematics anxiety, anger, and accomplishment. In Chapter 6, she introduces students to reasons for keeping a journal. These reasons are connected to overcoming anxiety and clarifying learning techniques, and understanding mathematical concepts and problems. In addition, she suggests seven ways students can use journals effectively.

Frankenstein, M., & Powell, A. B. (1989). Empowering non traditional students: On social ideology and mathematics education. *Science and Nature*, 9(10), 100-112.

The authors discuss sociological roots of underprepared mathematics students and how, among other vehicles, reading and articulating help students construct meaning in mathematics. Some articulation activities they discuss are free writing, journals or learning logs, and multiple entry logs. These activities enable learners to explore, reflect, and conjecture. The authors indicate that writing serves as an outlet for anxieties and other emotions that can hinder students' progress in mathematics. They also suggest that responses to student writings should be non judgmental and invite students to explore their ideas further.

Gopen, G. D., & Smith, D. A. (1989). What's an assignment like you doing in a course like this? Writing to learn mathematics. In P. Connolly & T. Vilar (Eds.), *Writing to learn mathematics and science* (pp. 209-229). New York: Teachers College Press.

In the context of a computer based calculus course, the authors describe how students' lab reports become learning tools. After instructors comment on the first drafts, students revise their reports, applying principles of "reader expectation," a writing approach that they learn in a university required writing course. By providing examples of student writings, the authors show that revising forces students to recast and refine their ideas into clearer and more precise form and, thereby, enable them to deepen their understanding of the concepts and techniques of the course.

Hoffman, M. R., & Powell, A. B. (1989). Mathematical and commentary writing: Vehicles for student reflection and empowerment. *Mathematics Teaching*, 126, 55-57.

The authors assert that writing in mathematics classrooms should not be limited to note taking. Instead, they suggest that writing activities should be generative, allowing learners to produce personal and unique mathematics. The activities should also enable students to acquire control, understanding, and a sense of security with mathematical ideas. To accomplish these ends, the authors describe two writing vehicles: mathematical and commentary. The former allows students to express themselves using mathematical symbols, while the latter prompts students to reflect on their doing, thinking, and feeling toward mathematics. Finally, the authors suggest a matrix for classifying the type of writing that students produce and, within it, which is most beneficial for learning.

Kenyon, R. W. (1987). Writing in the mathematics classroom. *New England Mathematics Journal*, (May, 1987), 3-19.

For Kenyon, writing is like problem solving. In this context, he discusses long term and short term assignments and notes, what to look for, what to expect, and how to respond to the initial writings of students.

Lax, A. (1989). They think, therefore we are. In P. Connolly & T. Vilar (Eds.), *Writing to learn mathematics and science* (pp. 249-265). New York: Teachers College Press.

After critiquing conventional pedagogies of mathematics and exploring why students suffer math anxiety, Lax proposes a different approach to teaching mathematics, one which is problem centered and engages students in writing while they work in groups and outside of class. She suggests that both students and teachers benefit and, using problems, explores questions that students may ask and ways of handling them. She also shows how students' previous experience with mathematics may determine the complexity of the questions they ask.

Layzer, D. (1989). The synergy between writing and mathematics. In P. Connolly & T. Vilar (Eds.), *Writing to learn mathematics and science* (pp. 113-121). New York: Teachers College Press.

Layzer supports the notion that writing about mathematical ideas helps learners acquire a deeper understanding of mathematical concepts. He correlates the ability to think and write clearly on a mathematical concept with a high level of understanding of that concept. For him, the clearer and more precise a learner's writing is, the deeper the understanding the learner has of the mathematical concept.

LeGere, A. (1991). Collaboration and writing in the mathematics classroom. *Mathematics Teacher*, 84(3), 166-171.

LeGere engages students in both collaborative and writing activities to aid learning and develop critical thinking. Of the writing activities she uses, she divides them into four categories: personal mathematics history, problem analysis, quiz taking, and open ended writing. For each category, she provides examples of writing activities and then discusses benefits.

McDonald, M. M. H. (1990). *Not just another class*. Washington, DC: The Mathematical Association of America.

McDonald discusses some of her reactions to being a student in a course on operations research that required journal writing on which the instructor commented. McDonald describes her initial anxiety about the course and difficulty in writing non narrative, thoughtful journal entries. Gradually, she begins to see

how writing enhances her learning and anxiety is replaced by confidence. McDonald cites additional benefits that she experienced from using journals and, in the context of journals, how the dialogue between students and an instructor allows an instructor to respond more readily to student needs.

Mett, C. L. (1987). Writing as a learning device in calculus. *Mathematics Teacher*, 80, 534-537.

For teaching different levels of calculus, Mett suggests appropriate writings activities and discusses their benefits.

Mett, C. L. (1989). Writing in mathematics: Evidence of learning through writing. *The Clearing House*, 62(7), 293-296.

Attempting to respond to requests for "proof" that writing helps students acquire a better understanding of mathematics, Mett discusses examples of student journals to demonstrate how these writings help students think about and understand and use new concepts and terminology in mathematics.

Mett, C. L. (1990). A writing project in operations research. In A. Sterrett (Ed.), *Using writing to teach mathematics* (pp. 87-92). Washington, DC: The Mathematical Association of America.

Mett investigates the use of writing in a course on operations research. Students develop projects and report them orally and in writing. She provides examples of student writings along with her responses. She also discusses ideas and questions that came up during and after her investigation and, as appendices, includes her syllabus and project schedule.

Miller, L. D. (1991). Writing to learn mathematics. *Mathematics Teacher*, 84(7), 516-521.

Miller responds to questions that teachers might pose about how writing improves learning, how to implement writing activities, how to handle the reading of student writings, should spelling and grammar errors be corrected, and whether writing will interfere with covering content. In answering these questions, she discusses the use of "impromptu writing prompts." She also suggests ways of getting started with writing prompts.

Miller, L. D. (1992). Tips for beginners: Begin mathematics class with writing. *Mathematics Teacher*, 85(5), 354-355.

Providing three examples of writing prompts, Miller advocates using prompts during the first five minutes of class to promote students' procedural and conceptual understanding of mathematics.

Myers, N. C. (1991). Writing and speaking to learn geometry. *Primus*, 1(3), 287-294.

Myers describes a writing to learn program in a geometry course. Students engage in writing activities ranging from mathematical autobiographies to proof writing and proof analyses. Using Reader Expectation Theory, they revise their first drafts, producing clearer and sharper writing, students deepen their understanding of geometry. Myers also discusses how she integrates writing into the curriculum and provides students' as well as her own evaluations of the course.

Powell, A. B. (1986). Working with "underprepared" mathematics students. In M. Driscoll & J. Confrey (Eds.), *Teaching Mathematics: Strategies that Work* (pp. 181-189). Portsmouth, NH: Heinemann.

In part of this article, Powell discusses three types of writing activities: (1) journals, (2) creative writing and research problems, and (3) explorations and problem-generating activities. He provides examples of student writings and suggests ways of responding to them.

Powell, A. B., Jeffries, D. A., & Selby, A. E. (1989). An empowering, participatory research model for humanistic mathematics pedagogy. *Humanistic Mathematics Network Newsletter* (4), 29-38.

The authors describe a research model that involves, as co-investigators, those who are traditionally subjects. In this participatory model, they use writing as a vehicle to engage students and instructor in interdependent learning and as a means of empowering students. The authors also suggest the relationship of their model to a humanistic perspective of mathematics.

Powell, A. B., & Lopez, J. A. (1989). Writing as a vehicle to learn mathematics: A case study. In P. Connolly & T. Vilardi (Eds.), *Writing to learn mathematics and science* (pp. 157-177). New York: Teachers College Press.

After noting the connections between reflecting on experiences and learning, the authors discuss writing as a tool to enhance reflection and distinguish efficacious writing activities, ones that involve students in producing mathematics. Using two of these writing activities—free writing and journals—the authors provide evidence of how, in a semester, writing enabled a student to develop insights and facility in mathematics. The student is one of the authors.

Powell, A. B., & Ramnauth, M. (1992). Beyond questions and answers: Prompting reflections and deepening understanding of mathematics using multiple entry logs. *For the Learning of Mathematics*, 12(2), 12-18.

While describing a writing to learn technique called multiple entry logs, Powell and Ramnauth highlight the ability of logs to prompt generative reflections on mathematical texts. They interpret and evaluate examples of logs that Ramnauth wrote after he began to see Powell for help in a precalculus class. They also discuss the benefits of multiple entry logs for teachers, tutors, and students.

Rose, B. (1989). Writing mathematics: Theory and practice. In P. Connolly & T. Vilaridi (Eds.), *Writing to learn mathematics and science* (pp 15-30) New York: Teachers College Press

Rose defines transactional and expressive writing and how they can be applied to the teaching and learning of mathematics. She discusses different types of expressive and transactional writing activities, relates these to research findings, and suggests pedagogical uses in these activities.

Rose, B. J. (1990). Using expressive writing to support mathematics instruction: Benefit for the student, teacher, and classroom. In A. Sterrett (Ed.), *Using writing to teach mathematics* (pp 63-72) Washington, DC: The Mathematical Association of America

Rose describes three expressive writing activities that she assigned to students in her calculus class: spontaneous dialogue journal writing, autobiographical narratives, and focused, in-class writing. She discusses how these activities benefit students, the instructor, and the classroom environment.

Stempien, M., & Borasi, R. (1985). Students' writing in mathematics: Some ideas and experiences. *For the Learning of Mathematics*, 5(3), 14-17

Stempien and Borasi discuss five activities to involve students in writing to learn mathematics: schematic reports, stories, essays, diaries and anecdotes, and dialogues. They provide examples of these activities, using the writings of high-school and college students, and describe difficulties and benefits of the activities.

Tobias, S. (1989). Writing to learn science and mathematics. In P. Connolly & T. Vilaridi (Eds.), *Writing to learn mathematics and science* (pp 48-55) New York: Teachers College Press

Tobias advocates the use of writing in learning science and mathematics. She examines how students and professionals use writing as a learning tool and describes two activities: Minute Papers about Muddiest Points and The Divided Page Exercise.

Van den Brink, J. (1987). Children as arithmetic authors. *For the Learning of Mathematics*, 7(2), 44-47

After raising questions about whether children could be writers of arithmetic textbooks, the author presents the work of first grade students who write an arithmetic book for future first graders. Using their everyday experiences, students create problems that illustrate addition. In some of the problems, the author discovered that they incorporated skills learned in kindergarten and that their writings revealed not only what they learned but also how they were taught.

White, D. L., & Dunn, K. (1989). Writing and the teacher of mathematics. In P. Connolly & T. Vilaridi (Eds.), *Writing to learn mathematics and science* (pp 95-111) New York: Teachers College Press

After speaking to teachers, the authors learned that many wanted to teach problem solving as a component of their course but could not because they lacked an effective pedagogical method. Thus, they initiated MATHCAPS, a project designed to introduce teachers to different problem solving strategies and ways to teach them. A major component of the project was writing, which became a tool for teachers to reflect on teaching strategies and mathematical concepts. In the process, it served as a catalyst to change their beliefs about teaching and mathematics. This change, from a rigid, algorithmic computational focus to a more conceptual focus helped the participating teachers improve their teaching and learning of mathematics.

Williamson, M. M., & McAndrew, D. A. (1987). Writing in college developmental mathematics. *Research & Teaching in Developmental Education*, 3(1), 14-21

The authors advocate informal, longer, and non-graded writing activities for learning mathematics. They describe activities using learning logs and propose ways that teachers can monitor and respond to student writings.

Worsley, D., & Mayer, B. (1989). *The art of science writing*. New York: Teachers & Writers Collaborative

Though this book is mainly about science writing, Worsley and Mayer do treat writing in mathematics, contending that it is essential for grasping concepts as mathematicians know them. Two useful sections are "Questions and answers," where they treat issues that attend the uses of writing in mathematics teaching, and the appendix, "Using writing in mathematics," written by R. W. Kenyon, where he suggests writing assignments, grouped into two categories: eight short-term activities and seventeen long-term projects.

## Notes

<sup>1</sup> For another example in the field of mathematics education, specifically in the area of critical mathematics literacy involving working class adults, see Frankenstein (1992)

<sup>2</sup> For details on this use of journals, see Powell & Lopez (1989). Also, at the end of this article, the annotated bibliography, pages eighty-one to eighty-nine, contains references to other pedagogical uses of journals.

<sup>3</sup> Students of the Newark College of Arts and Sciences, Rutgers, the State University of New Jersey, need only to demonstrate proficiency in college algebra.

<sup>4</sup> See Myers (1991) and Gopen & Smith (1989 or 1990) for details on 'reader expectation theory' as well as for discussions on its application to writing to learn mathematics.

<sup>4</sup> Geeslin, in 1977, published the first work we found specifically on writing to learn mathematics; and after 1987, at their annual meetings, the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) included numerous sessions on the topic. Furthermore, in *Curriculum and Evaluation Standards for School Mathematics*, published in 1989, the NCTM emphasizes the importance of communication in learning mathematics, while, in 1990, the MAA published Sterrett's edited collection, *Using writing to teach mathematics*, aimed at instructors of collegiate mathematics.

<sup>6</sup> One can extend this instrumental notion of writing and mathematics to include the ordering of both thoughts about effect and thoughts directed toward monitoring effect. For a discussion of this point, see Powell & Lopez (1989).

<sup>7</sup> Transactional writing uses language "to get things done" to inform people (telling them what they need or want to know or what we think they ought to know), to advise or persuade or instruct people. "We use it whenever we need an "accurate and specific reference to what is known about reality." Expressive writing is "thinking aloud on paper." It has the function of revealing the speaker, verbalizing his consciousness. [It] submits itself to the free flow of ideas and feeling." (Britton *et al.*, 1975, pp. 88-90).

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**Carlos Ramos** is currently pursuing a baccalaureate degree in mathematics and minoring in history at Rutgers University, the Newark College of Arts and Sciences. In 1992, he received a Henry Blumenthal Scholarship. **Esaie Pierre** is pursuing a baccalaureate degree in biology and minoring in mathematics at Rutgers University, the Newark College of Arts and Sciences. In 1990, he was selected to participate in the Minority Biomedical Research Support Program. **Arthur B. Powell** teaches mathematics and mathematics education at Rutgers University in Newark. Among other projects, he co-edits the newsletter of the Critical Mathematics Educators Group.

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