Seeding Ethnomathematics with *Oware: Sankofa*

The word *sankofa*, as used by the Akan-speaking people of Ghana, is a grammatical imperative, meaning that to advance, one must reflect on and reclaim traditional cultural ideas. Reflecting on mathematical ideas expressed in cultural products and practices is an important part of ethnomathematics (D'Ambrosio 1990; Gerdes 1999). As Powell and Frankenstein (1997) suggest, ethnomathematics emerges from discourse about the "interplay among mathematics, education, culture, and politics" (p. 5). In classrooms, ethnomathematics can be implemented by investigating the mathematics of cultural products and practices, such as games, with people from that culture or by exploring the mathematics of a different culture to help students enrich their construction of mathematical ideas (Powell and Frankenstein 1997, p. 249).

Arthur B. Powell and Oshon L. Temple

Children are naturally attracted to, and motivated by, games. Games that involve number or strategy stimulate children's mathematical imagination and thinking. Developmental psychologists and mathematics educators have noted that children are intellectually motivated to learn through games and other thinking activities (Barta and Schaelling 1998; Ginsburg 1989; Salvadori and Wright 1998). Games often foster self-directed exploration. When children feel appropriately challenged by a game, they become intrinsically motivated to discover "the secret" of winning or of avoiding loss. They "decide to practice at certain times . . . attempt to expand their knowledge . . . [and] . . . request information when they believe it is needed" (Ginsburg 1989, p. 84). The sheer pleasure of playing a particular game enables children to learn the mathematical ideas embedded in it as a by-product of playing. Moreover, by actively observing and carefully listening to children as they play, teachers can learn about how children think and the mathematical ideas that they are constructing.

Board games are also important cultural instruments for engaging children in intellectual explorations that embody interesting and rich mathematical structures. While playing games, children establish intellectual frameworks that enable them further to construct and comprehend complex mathematical ideas, strategies, and theories. Through the work of ethnomathematicians (see Barta and Schaelling [1998]; Eglash [1999]; Gerdes [1999]; Ismael [1996]; Powell and Frankenstein [1997]) and those interested in multicultural mathematics (e.g., Bell and Cornelius [1988]; Lumpkin and Strong [1995]; and Zaslavsky [1986, 1994, 1996, 1998, 1999]), teachers in the United States have seen how games played in other cultures can help children in our society hone mathematical skills, thinking strategies, and problem-solving abilities and develop an awareness of their participation in a global community.

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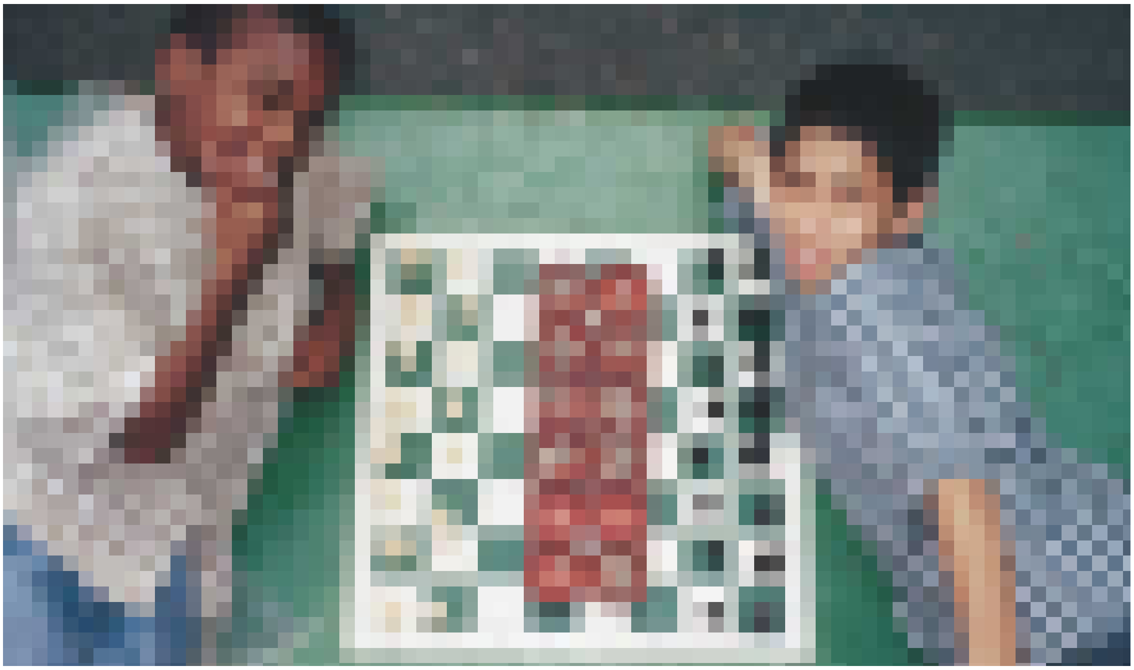


One such game is *oware*, an African board game that provides rich opportunities for all children to build and extend arithmetical ideas and strategic thinking and to explore important social behaviors. Moreover, introducing oware, either in or out of school, can help children understand that humans encode their mathematical ideas in diverse cultural products, including architecture, art, games, music, written texts, and so on. Children who play oware not only build mathematical ideas but also interact with aspects of African culture.

People play oware and similar games throughout Africa and in other parts of the world. The history of the game reaches back to antiquity. Boards cut into stone have been found in many parts of Africa, some dating to about 1580 B.C. (Bell and Cornelius 1988, p. 24). In our work, we engaged a group of African American and Latino children in playing oware. We selected this game for two reasons. First, children have a common biological heritage rooted in Africa (Diop 1974; Tattersall 1997). Second, children are cognitively enriched when they encounter and appreciate diverse cultural manifestations of mathematical ideas. The children in our group, ages six to twelve, learned to play the *Abapa* version of oware (Agudoawi 1992). They attended a day camp at the Ravenswood Community Center in Long Island City, New York. Most of the children reside in one of two public housing projects, Ravenswood or another one nearby.

Abapa is one of many versions of oware played in Ghana, West Africa. In the United States, the game is usually called *mancala* and is available commercially from a number of sources, ranging from African craft and gift shops to toy stores and mail-order catalogs. For both an excellent book and a PC-based version of the game, see Kovach (1995). The book contains historical and cultural information, including important early papers on oware by mathematicians and anthropologists. Internet users can find other Web sites to learn about and play oware, including **imagiware.com/mancala/**. To view boards from different parts of the world, check out **www.ahs.uwaterloo.ca/~museum/vexhibit/CC/countcap.html**. For the rules in Spanish, visit **members.tripod.com/~juegosdetablero/wari.html**. To download a Macintosh version of oware called Stones II, go to **www.uoguelph.ca/~ntagg/stones.html**.

Rules of Oware

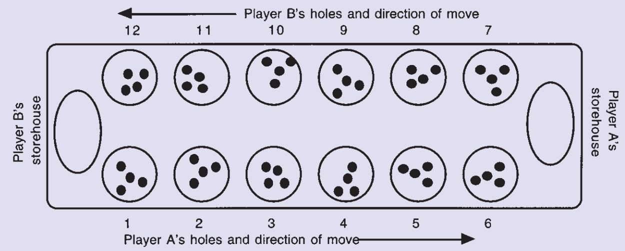
The object of oware is to accumulate the highest number of harvested pieces, twenty-five or more. The player or team that does so wins and moves first in the next game. The overall winner is the player who wins three games in succession.

To play oware, players sit facing each other with the gameboard placed lengthwise between them. Each player places four pieces, or seeds, in each of six holes on his or her side of the board, and the players decide who begins first (see **fig. 1a**). Player A scoops up all the seeds from any one of the six holes on his or her side; beginning with the next hole, player A distributes, or sows, one seed in each successive hole counterclockwise around the board until all those seeds have been sown. Players do not sow seeds in either storehouse. During the game, players may count the seeds in any hole visually but not deliberately. If a player accumulates twelve or

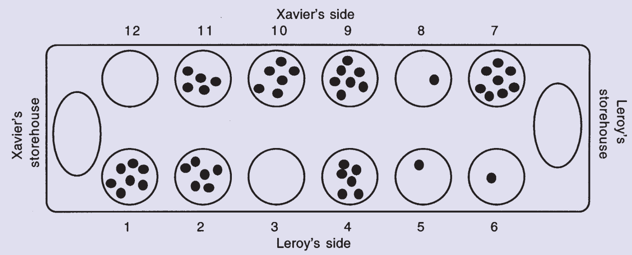
FIGURE 1

Moves and strategies of the game

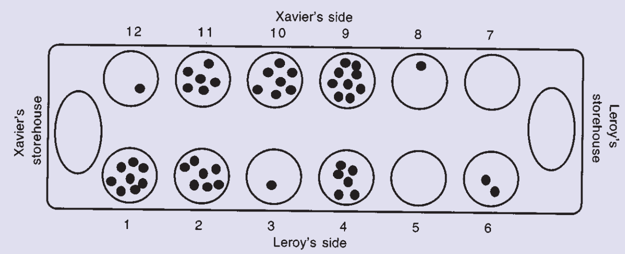
Move Pairs			
Leroy	Xavier	Commentary	Illustration
1. 6–10	12–4	Leroy takes seeds from hole 6 and sows one seed in each hole around the board to hole 10. Xavier responds by playing hole 12 and sows seeds to hole 4. In each move, the player sowed four seeds.	(a)
2. 5–9	8–2		
3. 6–7	12–1		
4. 3–8	8–9	Leroy moves from hole 3 to hole 8. This move threatens Xavier's hole 8, which Leroy could harvest using his hole 1 or 2. Xavier's move from hole 8 to hole 9 prevents Leroy from harvesting hole 8 in the next move by playing either 1x8 or 2x8 (x denotes a harvest).	(b)
5. 5–6	7–3		
6. 6–8	9–6	Leroy plays hole 6, harvests the two seeds from hole 8, then places them in his storehouse, which is the one to the right. Xavier plays hole 9 and sows to hole 6. This response shows that Xavier may not be thinking more than one move ahead.	(c)
		Leroy and Xavier's moves typify those of novice players. In (c), Leroy can harvest hole 8 using hole 6. From the position in (d), Xavier plays 9–6, which enables Leroy to harvest hole 7 in the next move. Xavier could have prevented Leroy from harvesting any holes had he played 12–1. From the position in (e), Leroy captures defensively by playing 6–7, avoiding Xavier's possible captures of 10–6 or 11–6.	(d), (e)



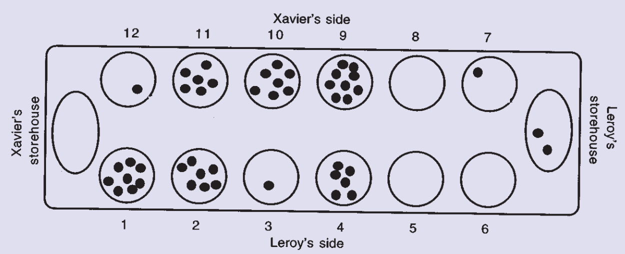
(a) Oware gameboard in initial setup with four seeds in each hole



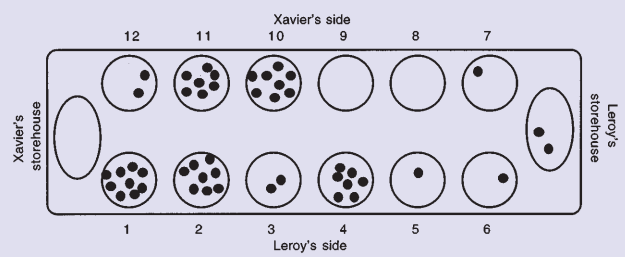
(b) Oware gameboard after Leroy's fourth move (3–8). Leroy's holes 1 and 2 threaten Xavier's hole 8.



(c) Gameboard before Leroy's sixth move



(d) Gameboard before Xavier's sixth move



(e) Gameboard before Leroy's seventh move

Note: The symbol – denotes a move.

more seeds in a hole, then he or she must skip that hole in coming back around. If the opposing player, player B, has no seeds in his or her holes, player A must, if possible, make a move that seeds the opponent for the sake of continuing the game. Doing otherwise causes player A to forfeit the game.

A player may capture seeds only from his or her opponent's side of the board, placing harvested seeds in the storehouse. A harvest occurs when, for example, player A's last seed drops into one of player B's holes that contained one or two seeds but now contains two or three. Player A harvests the contents of the hole and places the seeds in his or her storehouse. Player A also harvests the contents of those holes that contain two or three seeds and come immediately before the harvested hole, with no intervening holes that contain fewer than two or more than three seeds. Player A may not harvest more than five holes at a time. If player A harvests all six holes, he or she forfeits the game because player B would have no seeds with which to play.

The game ends when player B, for example, has no seeds left on his or her side of the board and player A cannot sow one or more seeds into B's holes. The pieces remaining on the board are added to the winner's captives.

Excerpts from an Oware Game

To illustrate the classroom potential of oware, we present three moves of a game played between Leroy Jenkins, age 9, and Xavier Saez, age 11, and discuss a problem-solving strategy that we observed in their play. (See **fig. 1.**) The initial setup of their game is the same as the one shown in (a) in **figure 1.** We use a notational format adapted from chess and alongside it, present a commentary on each pair of moves. In our notational format, we assign each hole around the board a number from 1 to 12, starting from the leftmost hole on the first player's side and continuing counterclockwise to the rightmost hole on the second player's side.

After thirty-nine moves, Leroy wins. In general, the combination of player A's move and player B's response constitutes one move pair of a game. In chess, one move of the pair is termed one *move-ply*. In these excerpts, Leroy and Xavier demonstrate thinking that is one and two move-ply deep. Children ultimately develop the ability to think three or more move-ply deep, which means to envision what the game state might be after three responses. Leroy and Xavier's experience with oware was short. Given more time, they would have been able to use developed strategies consciously and at greater depth. Their playing, largely without developed strategies, is common to novice players. With experience, children begin to formulate intermediate goals, learn to consider move options and the set of possible responses, and search for patterns in the configuration of seeds on the board to recognize a threat or an opportunity to harvest. As they do with move-ply depth, children develop these problem-solving skills over time.

Move-ply is not the only type of mathematical thinking that students can develop by playing oware. As children compete to win oware games, they must become involved in creating one-to-one correspondences, counting, developing effective counting techniques, and executing all basic arithmetical operations. When the number of seeds in a hole is greater than eleven, then knowing into which hole the last seed will fall engages the mind in performing clock, or modular, arithmetic. To provide more opportunities for children to develop these ideas, teachers can ask students to discuss and write about their playing strategies. The reproducible **template** that accompanies this article contains enough boards to record six pairs of moves, as well as space for reflection. Typically, more than one template is needed to record a single game.

Other Mathematical and Cultural Ideas of Oware

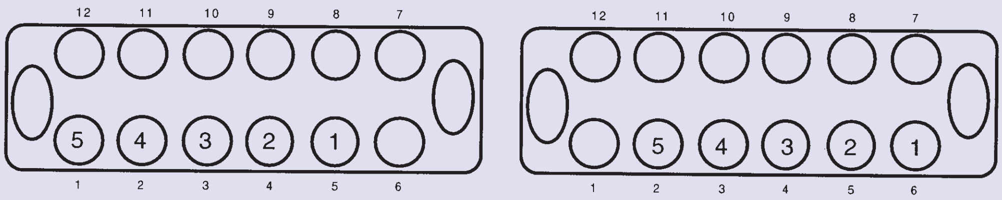
Following the intellectual path of Africans, players of oware can learn to recognize interesting and important numerical patterns and acquire insights into useful and sophisticated mathematical ideas. For example, Eglash (1999) reports that Ghanaian players refer to a particular self-replicating pattern as a "marching group" (p. 101). A marching group is a consecutive, decreasing sequence of seeds ending at a hole that contains one seed (for instance, 4-3-2-1). Imagine any sequence of seeds to be a snake heading counterclockwise around the board. Each move will involve playing the tail of the snake, or sequence. If the tail of a marching-group sequence is played, the pattern repeats itself (see **fig. 2**). This observation allows players to set up captures and remove threats. Eglash relates this self-propagating pattern to two interesting mathematical ideas: one-dimensional cellular automaton and triangular numbers (pp. 101–6).

Marching groups embody both these ideas. In any marching group, such as 5-4-3-2-1, the total number of seeds is a triangular number (see **fig. 3**). In this instance, 5 + 4 + 3 + 2 + 1 adds to 15, the fifth triangular number. Furthermore, regardless of how a triangular number of seeds is distributed among consecutive holes, a player encounters a marching group by playing the tail of the sequence a certain number of times. Consider the following example using the fourth triangular number, 10. If

FIGURE 2

"Playing the tail of the snake"

Playing the tail of the sequence just one time causes the marching group 5-4-3-2-1 to repeat itself. The sum of the seeds in the five holes is the fifth triangular number, 15.



ten seeds are distributed, two into each of the first five holes of an oware board, 2-2-2-2-2, and the tail of the sequence is played six times, the player will encounter the marching group 4-3-2-1 (see **fig. 4**). This pattern introduces the idea of a one-dimensional cellular automaton.

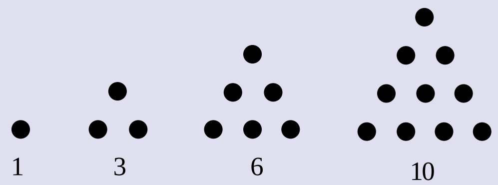
Other rich explorations can emerge from investigating numerical patterns and playing situations in oware. For example, students might try to determine the maximum number of iterations, or repetitions, needed to reach the marching group for a given triangular number of seeds distributed among consecutive holes. In general, oware games can involve extensive analysis, a task that is made easier by investigating controlled situations from the perspective of one player. Teachers might ask, for example, "What sequence of moves allows the first (or second) player to make the earliest possible harvest?" Even without such investigations, early experience playing oware can provide both visual and tactile foundations for more formal explorations of certain mathematical ideas, including figurate numbers and their properties, iterative and self-referential processes, and cellular automata.

Games reveal the thoughts and lives of those who invent them. The physical structure and materials, as well as the rules of a game, reflect the culture that created it. As a result, when students play a game such as oware, they interact with aspects of the culture in which it originated. For instance, not all games involve opposing sides, notions of attack and defense, or rules for capture. Similarly, one cannot play every game with dried seeds and in hollows scooped in the ground or on ornately carved boards. Traditional African values, such as sharing and saving face, are manifested in two rules of oware: (1) a player must seed an opposing player when the opponent has no seeds on his or her side of the board and (2) a player may not win by mercilessly capturing the seeds in all six holes at once. The rules of play and harvest compel players to treat one another politely and with dignity. Nevertheless, among players and onlookers, sarcastic remarks are part of the amusement of play (Odeleye 1977, p. 14). **Figure 5** summarizes some of the mathematical and other cultural ideas embodied in the game of oware.

FIGURE 3

Two representations for the first four triangular numbers

Numerals and geometrical arrays of dots each form a triangle, allowing a single dot to stand for the first triangular number.

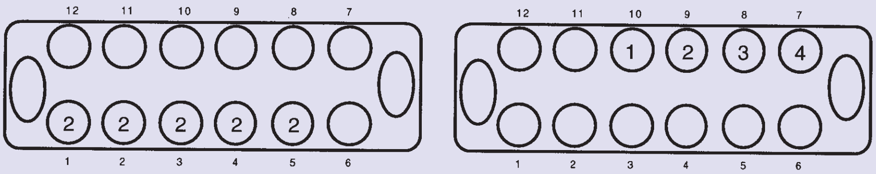


In addition to mathematics, oware also offers connections with other curriculum areas. As an art project, students could create their own oware boards or variations of traditional ones. To create a board, use an empty egg carton, preferably a two-row, nonpolystyrene carton that holds exactly one dozen eggs; forty-eight counters, such as large dried beans or chickpeas, colored beads, small glass counters, or marbles; and two small paper or plastic cups or dishes, each capable of holding three-fourths of the forty-eight seeds. Students could analyze actual gameboards or ones pictured in books for mathematical and cultural information. Useful for this purpose is de Voogt (1997), which contains photographs of more than fifty beautiful oware boards. This book catalogs the British Museum's collection, likely the world's most extensive, of oware, or mancala, boards variously made of wood, cow dung, ivory, and corrugated iron, totaling 105 boards from Africa, Asia, and the Americas. Further possibilities for these kinds of investigations arise as children learn more than one version of the game.

These and other curricular explorations of oware form a link to the notion of *sankofa* and remind us that students of African descent have a rich cultural

FIGURE 4

A linear representation of the six iterations of playing the tail of the sequence



(a)
Initial distribution of ten seeds

(b)
Six iterations yield a marching group

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Ten seeds are distributed by placing two into each of the first five holes of an oware board. (Here, we have represented the seeds in one line.) After six iterations of playing the tail of the sequence, the original configuration is transformed into the marching group 4-3-2-1. The ten seeds appear in each configuration, and their sum equals the fourth triangular number.

FIGURE 5

Mathematical and cultural ideas embodied in Oware

Mathematical Ideas	Other Cultural Ideas
• Arithmetic operations	• Cooperation
• Counting techniques	• Competition
• Decision making	• Respect for others
• Inequality	• Self-control
• Modular, or clock, arithmetic	• Sharing
• Move-ply	• Teamwork
• Number line	• Planning
• Strategic thinking	
• Estimation	
• Number patterns	
• Self-replication	

heritage. All children and teachers can benefit from oware's many important mathematical and cultural values. In his review of Sapient Software's Oware!, teacher and scientist Irving Adler, well known for his books *What We Want of Our Schools* and *Mathematics and Mental Growth*, comments on the benefits that children experience from playing oware. He writes that children's "own experience with this African game that stresses mathematics, analytical skill and planning ahead serves as a potent refutation of racist stereotypes about Africans and Blacks" (**www.crl.com/~rkovach/oware/reviews.htm**). The educational benefits of incorporating oware and discussing its cultural history in mathematics classrooms are compelling and ought not to be overlooked.

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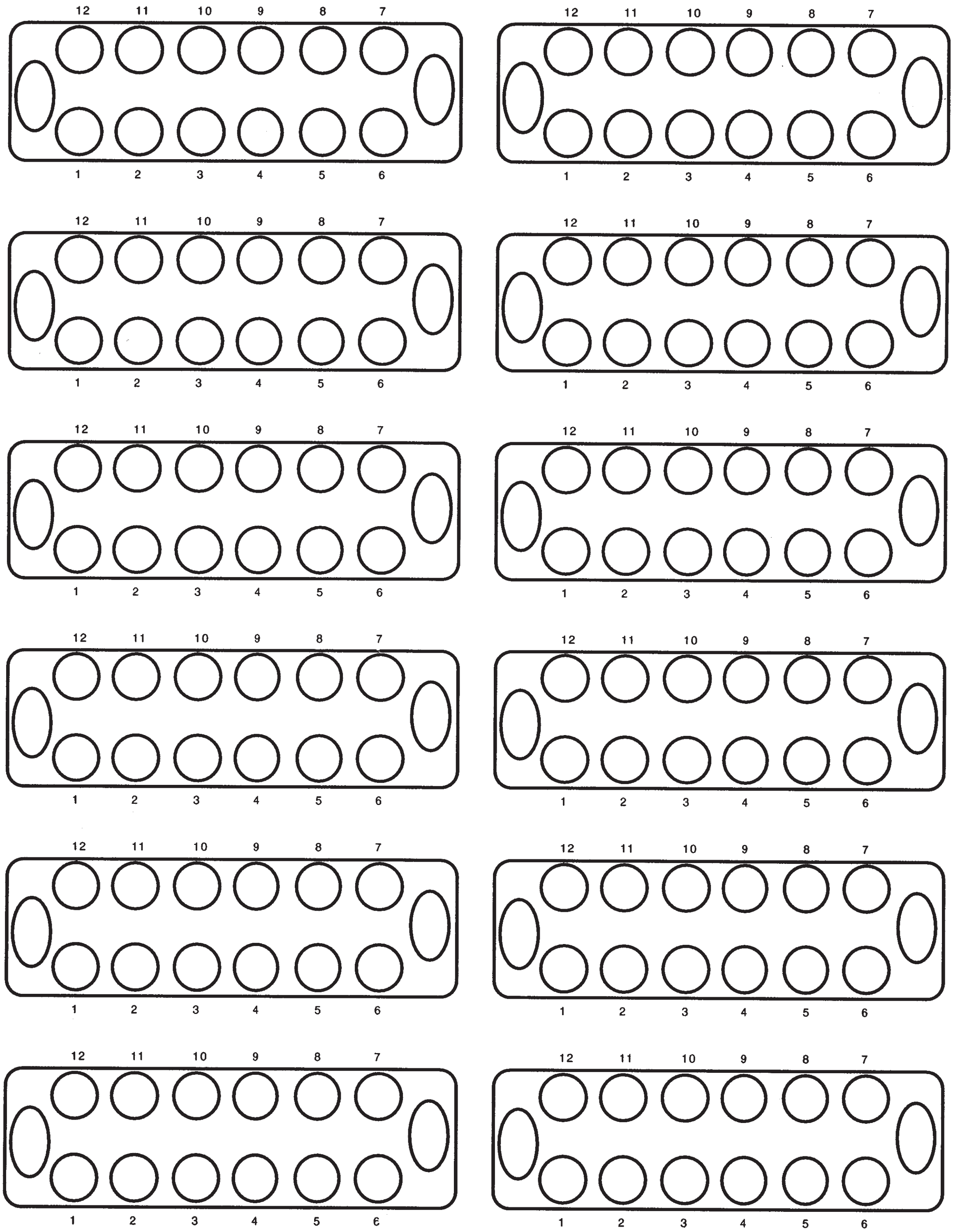
Move	Player A	Player B



Reflections:

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