Respecting Intellectual Diversity: An Ethnomathematical Perspective

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Introduction: Teaching Respect for People's Intellectual Activities

Stephen I. Brown has contributed much to a humanistic perspective on mathematics education. His concerns for the intellectual and emotional as well as ethical dimensions of the whole person engaged in mathematics intersects with our concerns for respecting the intellectual diversity of mathematical ideas among individuals and social groups and for the need to connect these concerns to social justice. Brown (1997; 1984) poses important questions such as how does mathematical thinking relate to areas of life beyond mathematics and what are the emotional experiences involved in doing mathematics (1997, p. 37) as well as what is the purpose of solving particular mathematical problems and how are relationships of mathematics to society and culture illuminated by studying how individuals view historically the phenomenon in question (1984, p. 13). Brown makes clear that an interdisciplinary, real-world mathematics curriculum must attend to humanistic as well as scientific problems: "I know of essentially no 'real world' problems that one decides to engage in for which there is not embedded some value implications" (1984, p. 13). Moreover, Brown pushes mathematics educators toward a fundamental respect for intellectual diversity by calling attention to the need to engage learners in considering "not merely complementary, but incompatible perspective on a problem or series of problems" (1984, p. 11). He ventures further and criticizes mathematics curricula for not enabling students to "*appreciate* irreconcilable differences rather than to resolve or dissolve them" (1984, p. 11).

In our view, to realize Brown's educational perspective it is important for educators to begin by respecting their own intellectual activity and those of their students. For us, one of the most important prerequisites for learning is respect.[1] Not only must teachers respect their own and their students' knowledge, but students often need to learn to respect their own and others intellectual activities and products. There are many different ways, using the intellectual activities of peoples all over the world to enable students to develop this respect. The basic idea behind all of them is that peoples' cultural material context influences their knowledge and activities in the world. In particular, students learn the importance of respecting their own and other peoples' knowledge, and how all of our cultural backgrounds interact with the development of mathematical knowledge. Further, the examples below show a variety of forms in which mathematical knowledge can be encoded differently from that in 'academic' textbooks.

[1] Shulman (2003), a lawyer who focuses on work-related issues, emphasizes the importance of respect in her book, *The Betrayal of Work: How Low-Wage Jobs Fail 30 Million Americans*. She presents a case study illustrating how lack of intellectual respect was the catalyst for a group of African American women, certified nursing assistants in rural Alabama, to organize a union: "none had ever gotten a raise of more than 13 cents. Some who had been there ten years were still making $6.00 an hour. But it was the lack of respect from their employer that motivated these women. They would tell their supervisors something important about patients but, they said, no one listens" (p. 3).

One example of how to develop this kind of intellectual respect is to ask students to think of other explanations for the well-known anecdote Ascher and Ascher (1997, p. 29) present about a trade between an African Demara sheepherder and an explorer. The herder agrees to accept two sticks of tobacco for one sheep but becomes confused and upset when given four sticks of tobacco for two sheep. The story was originally told with a racist sub-text that the African herder was so stupid he could not comprehend that 2 + 2 = 4. An alternative interpretation, respecting the herder's knowledge, "raises the issue of the difference between a mathematical concept and its application" (1997, p. 29) since sheep are not standardized units. So it is logical that a second, different animal would not also be worth two sticks of tobacco. "[T]he applicability of even the simplest of mathematical models becomes a question of cultural categorization" (1997, p. 29).[2] Zaslavsky (1973/1999, p. 32) relates that Howard Eves concludes his discussion of this anecdote in his book, *In Mathematical Circles* (9°), where he observes

> Yet these Demaras were not unintelligent. They knew precisely the size of a flock of sheep or a herd of oxen, and would miss an individual at

[2] Zaslavsky (1973/1999) relates that Sir Frances Galton first told this tale after he visited Africa. Galton, who coined the term "eugenics" in 1883, considered measurement "the primary criterion of a scientific study" (Gould, 1981, p. 75). In essence, he tried to "standardize" anything that might possibly be measured, including prayer, beauty, and boredom—the latter by "counting the number of [a person's] fidgets"(Galton, 1909, p. 278; as quoted in Gould, 1981, p. 75). He further believed that nearly everything he could measure was inheritable. When his cousin Charles Darwin pointed out that "men did not differ much in intellect, only in zeal and hard work," Galton countered that "the aptitude for work is heritable like every other faculty" (Galton, 1909, p. 290; as quoted in Gould, 1981, p. 77). So, it is not surprising that Galton could not see a more sophisticated reason for the sheep herder's confusion. Moreover, an important note for the politics of knowledge is that Galton was considered a leading intellect of his time and his "scholarship" had significant influence on the development of modern statistics (Gould, 1981, pp. 75 and 77).

once, because they knew the faces of all the animals. To us, this form of intelligence, which is true and keen observation, would be infinitely more difficult to cultivate than that involved in counting.[3]

Students also speculate that a contract for the first sheep must be renegotiated when the initial terms change (i.e., a second sheep is added to the deal). When students reflect on this example, they begin to develop respect for their own logic, and a respectful attitude of interest to learn from others about how they think and why they give the answers they do.[4]

Other examples from around the world that can be used to broaden students' respect for diverse knowledge and the interactions of culture and mathematical knowledge are:

- The French Basque concept of (mathematical) equality "evidenced by a variety of different circles and cycles." For instance, the giving and receiving of bread involves H_i giving to H_{i+1} and "the giving moves around the circle made up of all the households in the community. . . . The overarching concept, 'equal-equal,' is not a static relationship as is our

[3] Another example of this kind of observational knowledge is given by Bok (2003) in his account of his years in slavery in the Sudan. He describes how he was forced to herd sheep and how his captor "had a way of looking at the animals and knowing that they had not had enough to eat. When he found out that I had cut a corner, he beat me" (Bok, p . 44).

[4] Another example that Ascher and Ascher discuss is from Lévy-Brühl's 1910 work in anthropology, *How Natives Think*. He felt that the occasional substitution of 3, 7, or 9 for each other in the Indian Veddic religion, rituals, and legends was "an absurdity in logical thought…quite natural to prelogical mentality, for the latter, preoccupied with the mystic participation, does not regard these numbers in abstract relation to other numbers, or with respect to the arithmetical laws in which they originate" (1997, p. 31). Ascher and Ascher counter his conclusion with information from a recent field study of the Kédang who also use this kind of number substitution: "When used in symbolic contexts, odd numbers are associated with life and even numbers with death. Substitutions within these classes are possible if circumstances require it. If, for example, a ceremonial period of four days is stipulated but cannot be met, 2 days will do but 3 would be a serious infringement…The formation of these equivalence classes is an example of an abstract idea about number " (1997, p. 31).

conventional mathematical or everyday equality. It is a dynamic process of interaction in which an essential feature is that the participants know what is expected of them and they know what to expect from others. That is, the actors in the process move in synchronization, doing different things, at different times, but together making up a whole. If one were to stop the process at an arbitrary point in time, there would be inequities in what has been contributed, what has been received, and who is superior to whom. But, just as a circle is enclosed by a never-ending line, the process of creating an equal-equal relationship continues throughout the season and throughout the years." (Ascher, 1998, pp. 26-27)

- The Navajo concepts of space, discussed by Schulman (1994) in her summary of Bradley, Basham, Axelrod, and Jones' (1990) research about language and its impact on mathematics learning among various nations of Native American students:

> The authors observed that 'the Western world developed the notion of fractions and decimals out of a need to divide or segment a whole. The Navajo worldview consistently appears not to segment the whole of an entity.' (Bradley et al., 1990, p. 8) Among their conclusions are suggestions that 'non-Euclidean geometry, motion theories, and/or fundamentals of calculus may be naturally compatible with Navajo spatial knowledge. Math classes should begin with these notions and continue de-emphasizing the segmentation of notions into smaller parts.' (Bradley et al., 1990, p.

> 8) In other words, for some students, it might be appropriate to teach calculus as elementary mathematics, and fractions in college! (B. J. Shulman, 1994, p. 9)

- The Australian Aboriginal practice of counting related in Harris' (1987) discussion of how even the basic act of counting is laden with cultural influences (quoted in Roberts (1998)) where he observed an Aboriginal woman counting in her own language by dividing a set of pebbles as though they were turtle eggs. Harris concludes his research by stating that, 'Aboriginal people in this context saw division (sharing) rather than addition (accruing) as the essential role of counting.'
- The Australian Aboriginal use of music to encode geographic knowledge recounted by Chatwin (1987) summarized by his companion who stated that "Music is a memory bank for finding one's way about the world." (p.108). Chatwin details how although most Aboriginal Nations speak the language of their immediate neighbor, they do not speak the language of more geographically distant Nations. And yet, they can understand what land is being sung when they hear songs in *any* Aboriginal language.

> Regardless of words, it seems the melodic contour of the song describes the nature of the land over which the song passes. So, if the Lizard Man were dragging his heels across the salt-pans of Lake Eyre, you could expect a succession of long flats, like Chopin's 'Funeral March.' If he were skipping up and down the MacDonnell

> escarpments, you'd have a series of arpeggios and glissandos, like Liszt's 'Hungarian Rhapsodies.'
>
> Certain phrases, certain combinations of musical notes, are thought to describe the action of the Ancestor's feet. One phrase would say, 'Salt-pan'; another 'Creek-bed,'…'Rock-face' and so forth. An expert songman, by listening to their order of succession, would count how many times his hero crossed a river, or scaled a ridge—and be able to calculate where, and how far along, a Songline he was. (p.108)

- A contemporary academic mathematician's experience learning conceptually different proofs from his students. Henderson (1996) provides examples of the many geometric proofs he learned from his students, such as the theorem that stereographic projection from a sphere to a plane is conformal, and presents an analysis of why "the most memorable mathematics [he has] learned is from students who differ from [him] (a White man) in gender or race." He argues that persons who differ the most from him in terms of cultural background and gender "are most likely to have different meanings and thus have different why-questions and different proofs"(p.49). His perspective on the interactions of culture and the development of mathematical knowledge is to define "multiculturalism as listening to and learning from others who come from different experiences" (p.50).

Eurocentric bias has denied the intellectual rigor of these kinds of knowledge, considering it to be 'childlike' and 'primitive.' Ascher and Ascher argue that "there is not one instance of a study or restudy that upon close examination supports the myth of the childlike primitive" (1997, p. 33). Their examples and the others that we have provided not only support this point but also reveal how false assumptions about the mathematical knowledge of others and lack of respect for the logic of others intersects with racism when one considers what counts as mathematical knowledge.

In the following, we present more detailed examples from our mathematics teaching that encourage respect for a diversity of ways of knowing and for a diversity of knowledge. This approach represents an ethnomathematical pedagogy, and in the next section develop a more theoretical discussion of an ethnomathematical perspective. Then we conclude with a more theoretical return to our original point about respecting intellectual diversity.

Teaching Mathematics with Respect for People's Intellectual Activities

Respecting people's intellectual activities and products influences teaching and learning activities. This means that it is important to consider the politics of knowledge and the interaction of culture in the development of ideas. Questions such as which knowledge and whose knowledge is considered legitimate are an integral part of learning mathematics in the examples that we discuss. Furthermore, questions concerning the interactions of culture with the development of mathematical ideas are also integral to the activities. These instructional considerations represent teaching from an ethnomathematical perspective.

The first example focuses on an approach to using a cultural product to more deeply explore a topic in a typical basic academic mathematics lesson. In particular, it emphasizes a curricular context that respects the cultural product. Especially since this product, the Incan *quipu*, will initially look like a macramé necklace to, at least, students in the United States of America, discovering its mathematical properties is a deep lesson in respecting knowledge that has an inscriptive format and code that is different from one's own schema for inscribing and coding knowledge. Further, since the *quipu* encodes place-value differently from our decimal notation, it helps students see how material culture and the circumstances of people's lives lead to different expressions of mathematical ideas, while it also deepens students' concepts of the meaning of that mathematical idea. Our classroom presentation contrasts to a simplistic 'folkloristic' way of using examples from other cultures where the cultural product is used as a brief introduction or conclusion to the 'real' mathematics lesson. Finally, this example also illustrates our contention that the broader historical context in which ethnomathematical knowledge was developed, suppressed or stolen, after imperial devastation, needs to be included in the curriculum.

The second example gives a much more detailed learning experience which moves from a broad review of various algebraic concepts in Ancient Egyptian culture to an application of those concepts in the contemporary cultural context of a USA algebra classroom. Finally, the curriculum unit returns to an exploration of the specific presentation in a papyrus of the original Ancient Egyptian mathematical idea underlying the teaching module. Students here deepen their understandings of the contemporary academic algebra, as well as develop a respect for the different uses of the concepts in the algebraic studies of the Ancient Egyptians. This and the *quipu* outline of lessons treat the mathematical ideas of other cultures with respect, showing the complexities of their knowledge, and deepening learners' own knowledge through an analysis of varying representations and solution methodologies.

Using the Incan *Quipu* to deepen understanding of place value

The *quipu* is Incan cultural product that represents a system for recording statistical information. It consists of a series of knots, colored cords that hang from a main cord resembling a macramé necklace. In a basic mathematics class, a fundamental aspect of the *quipu* can be used to reinforce and deepen the meaning of place value. We are careful not to present the *quipu* as a kind of "folkloristic," five-minute introduction to the 'real' mathematics lesson. Rather, the Inca *quipu* is considered in its material context, connected to the situation and culture in which it was developed, "showing the necessity of any given piece of calculation, measure or pattern for the particular society of which it was a part" (Singh, 1991, p. 21). To do otherwise "implies that such a culture is not credited the role as a decisive and organic frame for the lives of the individuals, but rather the role as a static and illustrative frame which mainly is of archaeological importance" (Mellin-Olsen & Holnes, 1985, p. 106).[5] Further, the reasons why these contributions have been hidden—reasons that involve issues such as racism, sexism, and imperialism—are also part of the context in which the mathematical aspects are discussed.

An instructional suggestion is to first show pictures of actual *quipus* and to ask students to speculate about the uses of the *quipu*. Then have them look at the schematic below, which shows part of a *quipu* (Ascher, 1983, p. 274) and figure out how that place-value system works and compare it to the system we use. Then you can discuss more information about *quipus*, such as the fact that the "colors of the cords, the way the cords are connected together, the relative placement of the cords, the spaces between the cords, the knots on the individual cords, and the relative placement of the knots are all part of the logical-numerical recording" that can communicate complicated records such as data in multi-layered matrix charts (Ascher, 1983, p. 269). You can also discuss how the Incas had no writing as we think of that form of communication and how the *quipu*

[5] In a conversation with us, Bob Lange, a physics professor at Brandeis, shared an experience which underlines how important it is for us to be aware of the particular lens through which we view other cultures. Even when that lens is respectful, it can be static and false. In order to give students and educators in Zanzibar an experience with computers based on their mathematical knowledge, Lange designed a computer program that plays their version of the game *oware*, *bao ki swahile*. He felt nervous about introducing the program for he did not want to violate the aesthetics of the beautiful board or the tactile pleasure of actually removing the smooth, rounded seeds. He spent a lot of time planning ways to have the computer serve as a mere adjunct to the actual presence of the board and seeds. Still, he was concerned that the players would find the computer inappropriate or offensive. Instead, from the first move they made, the students completely ignored the board and seeds. They educated Lange that for them the ancient African game of *oware* is not primarily about aesthetics, but about abstract strategies. In the context of high technology, they found the computer was an engaging, appropriate representation of their contests of reasoning. Lange, and Michael Savage of the African Forum for Children's Literacy in Science and Technology in Nairobi, are currently writing an article in which they will be discussing this and other observations that have arisen from this unexpectedly appropriate use of technology.

challenges the ways in which we think of that form of communication. Next, you can relate how the *quipu* played a decisive role in the very methodical, data-organized Inca civilization. The Incas kept track of resources, taxes, and other data in the vast territory under their imperial control[6] through the messages encoded in the *quipu*. A runner would tie the *quipu* around his waist and carry the message to another place, where the next runner took the *quipu*, and so on. The *quipu* developed in an ideal way for the needs of the Inca civilization—portable, compact, clear, and not likely to be destroyed along the strenuous journey. Finally, you can look more broadly at the larger picture of the Inca civilization of three to five million people, which existed from about AD 1400 to 1560 in Latin America. They built extensive road and irrigation systems; imposed a system of taxation involving agricultural products, labor, cloths, and other 'finished' products; and built storehouses to hold and redistribute agricultural products, as well as to feed their army as it moved. We think that it is important to provide a broad context, including the idea that European imperialism is a probable reason the mathematics of the *quipu* is not more extensively known: "Within about 30 years after the Europeans reached the Andes and 'discovered' the Incas, the Inca culture was destroyed" (Ascher, 1983, p. 269). Not including this aspect is a strong political silence that disrespects our common world history. Further, it disrespects students' abilities to analyze situations in their full complexity.

[6] It is important not to romanticize other cultures that we study—some, of course were/are very peaceable; others, like the Incas, had colonial/imperialist empires.

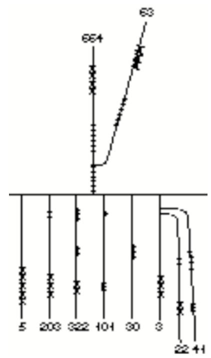

Figure 1. A representation of an Incan *quipu* with indication of the numerical value of each cord.

Using Egyptian algebraic and numerical-puzzle ideas to teach and deepen understanding of equations

In our next example, we use algebraic and numerical-puzzle ideas from ancient Africa for an algebra course module. It incorporates scholarship based on mathematical insights documented in an ancient Egyptian text. Unfortunately, the importance of Africa's contributions to mathematics and the central role of these contributions to the academic mathematics studied in schools have not received the attention and understanding that befit them.[7] Important projects to redress this state of affairs have been initiated by some historians (Diop, 1974; Jackson, 1970), historians of mathematics (Gillings, 1972/1982; Katz, 1998) as well as by scholars of ethnomathematics (Eglash, 1999; Gerdes, 1989, 1992, 1999; Joseph, 1991; Lumpkin, 1985, 2002; Lumpkin & Powell, 1995; Zaslavsky, 1973/1999). Documentary evidence of insightful and critical

[7] Diop (1974) argues a more general point about historiography: "The history of Black Africa will remain suspended in air and cannot be written correctly until African historians dare to connect it with the history of Egypt.…The ancient Egyptians were Negroes. The moral fruit of their civilization is to be counted among the assets of the Black world.…that Black world is the very initiator of the 'western' civilization flaunted before our eyes today. Pythagorean mathematics…and modern science are rooted in Egyptian cosmogony and science" (p. xiv).

algebraic ideas developed in ancient Egypt exists, but little of this information is taught to students studying mathematics, at any level.

The course module we have implemented contributes to redressing this lack of scholarship in school algebra courses by incorporating and appreciating mathematical ideas from the *Ahmose Mathematical Papyrus*[8] (Diop, 1985; Joseph, 1991; Lumpkin, 1985), now housed and displayed in the British Museum (except for a few fragments in the possession of the Brooklyn Museum).[9] It engages students in reflecting on their daily, informal experiences of performing (doing and undoing) physical and mental operations and relates these operations to algebraic techniques by building on the following three mathematical ideas present in algebraic equations extant in ancient Egypt and suggested in Problems 24 through 34 of the *Ahmose Mathematical Papyrus*:[10] (a) the concept of unknown or variable quantities (Boyer & Merzbach, 1989, pp. 15-16; Gillings, 1972/1982, p. 154); (b) undoing or, equivalently, inverse operations (Katz, 1998, p. 15); and (c) "think of a number" problems (Gillings, 1972/1982, pp. 181-184).[11] In what follows, these ancient algebraic ideas are used to develop a course module, the

[8] This text, found at Thebes in the ruins of a small building near the Ramesseum, is often referred to as the Rhind Mathematical Papyrus since, in 1858, Alexander Henry Rhind (1833-1863), variously described as a Scottish antiquary and Egyptologist, purchased it in Luxor . (Arnold Buffum Chace, Manning, & Archibald, 1927; Robins & Shute, 1987) The naming of such text is another example of how cultural imperialism obliterates the authorship of knowledge of the peoples it dominates. In this instance, the text is correctly named the *Ahmose Mathematical Papyrus* after the Egyptian scribe-scholar who authored it.

[9] The location of many important ancient African cultural products in Western museums, as well as those of other Third World cultures, evidences the endurance of cultural imperialism.

[10] In the preface to their book, Chace, Manning, and Archibald (1929) indicate that in the original papyrus the problems are not numbered and credit Eisenlohr with numbering them.

[11] Space does not allow us to elaborate on where and how each of these ideas represent themselves in the *Ahmose Mathematical Papyrus*; however, for such details, see Powell and Temple (2002; 2004).

last part of which returns to analyze specific problems presented in the *Ahmose Mathematical Papyrus.*

To engage students in developing an awareness[12] of the role undoing or inverse operations play in solving certain types of equations, one can present a type of "think of a number problem" as appears in Figure 2:

> "I'm thinking of a number. I subtract 11 from it, multiply by 3, and add 2. The result is 80. What's my number?"
>
> *Figure 2.* A "think of a number" problem.

This is an example of a "What's my number?"[13] problem that is likely to trigger in students an awareness of undoing that includes the ideas of using inverses: multiplicative and additive inverses or inverse operations as well as the reversal of the order of operations. Undoing is a mathematical process related to many aspects of daily life: dressing and undressing, wrapping and unwrapping parcels, or either physically or mentally retracing one's steps to find a misplaced or lost item, and so forth. In schools and textbooks, this quotidian process is

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[12] We use the term 'awareness' or 'mathematical awareness' in the technical sense suggested by Gattegno (1987) and elaborated on by Powell (1993, p. 358):

> Gattegno makes clear that, for learners, learning or generating knowledge occurs not as a teacher narrates information but rather as learners employ their will to focus their attention to educate their awareness. Learners educate their awareness as they observe what transpires in a situation, as they attend to the content of their experiences. As a learner remains in contact with a transpiring experience, awareness proceeds from "a dialogue of one's mind with one's self" about the content of that which one experiences (Gattegno, 1987, p. 6). One's will, a part of the active self, commits one to focus one's attention so that the mind observes the content of one's experience and, through dialogue with the self, becomes aware of particularities of one's experience. Specifically, in mathematics, the content of experiences, whether internal or external to the self, *can be feelings, objects, relations among objects, and dynamics linking different relations* [italics added] (Gattegno, 1987, p. 14).

[13] We use interchangeably the phrases "think of a number" and "What's my number?" problems.

often not related to mathematics but, of course, is fundamental to it. Resurrecting and rediscovering such links between quotidian and formal knowledge schemas is one project of ethnomathematics that involves respect for intellectual activities and connect to Brown's concern for relating mathematics to aspects of life beyond mathematics.

Students are asked to reflect on their undoing technique so as to specify completely its constituent processes and ideas. After students discuss their ideas, we introduce conventional terminology for labeling processes and ideas.[14] For the "think of a number" problem in Figure 2, students eventually might describe their process for solving it as follows:

> The first operation or action performed on the original number thought of was to "subtract 11." The second action was to "multiply by 3." The third and last action was to "add 2," and the result was "80." To undo the problem, we need to start at the end. Immediately before the result 80 was obtained, the last action performed was to add 2. To undo *add* 2 we can *subtract* 2 from 80, which is 78. Now, the action performed before 78 was obtained was to multiply by 3. To undo *multiply by* 3 we can *divide by* 3. So, we take 78 and divide it by 3, which gives 26. The action performed before 26 was obtained was to subtract 11. To undo *subtract* 11 we can *add* 11. So, the original number thought of was 37.

[14] This pedagogical approach is akin to the "five crucial steps in the Algebra Project curriculum process" (Moses & Cobb, 2001, p. 120). In both approaches, one begins with the experiential, mathematical world of learners and assists them to connect their ideas to the symbolic world of formal, academic mathematics.

The above description reveals an awareness that the mathematical ideas of inverse operations as well as reverse order of operations are useful for solving the “think of a number” problem given in Figure 2. This awareness is generalizable, and students use its generalization to handle more complex “think of a number” problems such as the one in Figure 3:

> “I’m thinking of a number. I raise it to the third power, add 8, raise it to the one-half power, add 1, and multiply it by 6. My result is 30. What’s my number?”

Figure 3. A “think of a number” problem involving integral and fractional exponents.

As the number and the complexity of the operations increase, students tend to develop *ad hoc* notational devices or inscriptions to represent “think of a number” problems. One notational device that we offer is circle equations (Hoffman & Powell, 1988, 1991). The circle equation for the problem in Figure 3 is given in Figure 4.

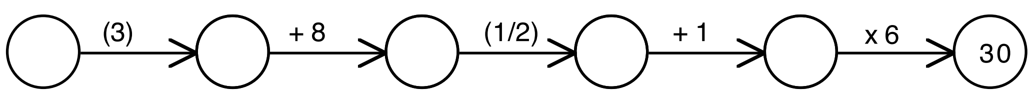

Figure 4: The circle equation of the “think of a number” problem given in Figure 3.

In circle equations there are several conventions. The first circle represents the number referred to by the phrase “I’m thinking of a number.” The expressions above the right-facing arrows correspond to the operators in the "think of a number" problem in the order in which they are mentioned. Finally, numbers placed inside parentheses are considered to be exponents.

Besides being a tool to represent graphically "think of a number" problems, circle equations can also be used, with slight modifications, to depict a process of solving such problems. Explicit in the structure of circle equations, read from left to right, is the order in which operations are performed in "think of a number" problems. Similarly, but reading from right to left, the process of undoing or solving can also be represented with left-facing arrows with inverse operations written below them.

Moreover, in the circle equation, the circle that stands for the phrase, "I'm thinking of number," can contain a variable, which means that the circle equation can be translated into the standard notation of academic mathematics (see Figure 5).

(a) $3(x-11)+2=80$ (b) $6\left\{\left(y^3+8\right)^{\frac{1}{2}}+1\right\}=30$

Figure 5. The standard notation for the "think of a number" problems given in Figures 2 and 4, respectively, (a) and (b).

These examples suggest that a variety of equations can be dealt with through an adaptation and combination of Ahmose's mathematical insights. Eventually, students are able to translate among standard and circle equations, and "think of a number" problems.[15]

[15] On the same point, Hoffman and Powell (1991) indicate that "with practice, pupils can imagine that a given equation is expressed as a circle equation and solve the equation, as it were, by sight" (p. 95). In this sense, then, circle equations assist learners to develop dynamic imagery of equations that allows them to solve equations without pencil and paper.

$$\left\{ \frac{\left[\left(4\left(r^{5}-2\right)+5\right)^{\frac{1}{3}}-1\right]!}{3}+1 \right\}^{\frac{1}{2}} - 2 = 1; \qquad 5\left(\sqrt{\frac{\left(\left(\log_5 y\right)-1\right)^{7}}{32}}\right) = 10$$

Figure 6. Two complex looking equations whose solution can be found using Ahmose's mathematical insights.

By working among these representations, students learn that equations such as the ones in Figure 6 simply appear complicated and are conceptually no more difficult to solve than an equation such as $3(x+6)=24$. The difficulty of the equations in Figure 6 arises from the amount of time required to solve them rather than any intrinsic complexity that they or their solution possess. In short, students learn that equations such as the ones in Figure 6 are, in fact, not difficult but only perhaps laborious. Moreover, once students develop facility solving equations like the ones in Figure 6, they deepen their algebraic understanding even further by, among a number of considerations, analyzing the use of circle equations to solve the following "think of a number puzzle" (Problem 28) from the *Ahmose Mathematical Papyrus*, as given in Gillings (1982, p. 182):

> Two thirds is to be added. One-third is to be subtracted.
>
> There remains 10.
>
> Make $\overline{10}$ [one-tenth][16] of this, there becomes 1. The remainder is 9.
>
> $\bar{\bar{3}}$ of it namely 6 is to be added. The total is 15.
>
> $\bar{3}$ of this is 5. Lo! 5 is that which goes out, and the remainder is 10.
>
> The doing as it occurs!

[16] Following Gillings (1972/1982), unit fractions are represented with a horizontal bar above the denominator and the fraction 2/3 is written with two horizontal bars over 3. This is an adaptation from the ancient Egyptian hieratic and hieroglyphic forms of writing fractions.

Let us state this in modern terms, adding a few clarifying details:

Think of a number, and add to it its $\bar{\bar{3}}$. From this sum take away its $\bar{3}$ [one-thirds], and say what your answer is. Suppose the answer were 10. Then take away $\overline{10}$ of this 10, giving 9. Then this was the number first thought of.

Proof. If the number were 9, its $\bar{\bar{3}}$ is 6, which added makes 15. Then $\bar{3}$ of 15 is 5, which on subtraction leaves 10. That is how you do it!

Figure 7. Problem 28 of the *Ahmose Mathematical Papyrus*.

Students successfully handling problems from the *Ahmose Mathematical Papyrus* such as the one in Figure 7 and complex-looking ones similar to those in Figure 6 increase their mathematical self-confidence. For they are able to solve equations that more advanced mathematics students initially find baffling. Students build and deepen their mathematical understanding and, thereby, own the knowledge they employ. They empower themselves while appreciating historical connections between the *Ahmose Mathematical Papyrus* and their own intellectual work. For instance, when students solve Problem 28, they encounter certain differences between it and the equations that they had been solving. In a circle equation, the unknown in Problem 28 needs to appear above the arrows and students may find it convenient to combine algebraic expressions to simplify what is contained in each circle. Students realize that at some point they must combine algebraic expression to find the number originally thought of. Also students become aware that they had been simplifying arithmetic expressions at each stage of solving a circle equation. Students then notice that problems such as this one illustrate that a numerical solution can provide a proof for whole class of problems. The *Ahmose Mathematical Papyrus* has other kinds of algebraic problems of varying complexity. Studying these problems provides opportunities for students to deepen their understanding by analyzing the complexity of problems and different solution methods, such as false position, Problems 24-27, and division, Problems 31 to 34 (Gillings, 1972/1982, pp. 154-159). Based on African algebraic techniques, students develop sophisticated mathematical insights and abilities and greater self-confidence as learners of mathematics. Since all students have a common biological heritage in Africa, they gain an increased appreciation and respect for the mathematical accomplishments of their ancestors and for the diverse cultural manifestations of mathematical ideas.

Moreover, students inquire into the politics of social structures that devalue the intellectual contributions of Africans and engage more deeply in academic mathematics. Egyptian algebraic knowledge has been devalued as being only practical and not employing proofs. As Gillings points out, examining the answers to problems they pose (for example, the answer to Problem 31 is $14\frac{28}{97}$) and even how many are posed abstracted from any measure make it clear that the problems were not posed to answer practical questions. Rather, as Gillings observes, "they were meant to illustrate one method for the solution of simple equations of this type, and although they did this, the simplicity of the method has been masked by the complexity of the unit fraction that arise in the process and by the unexpected operations to which the scribe was to forced to resort" (1972/1982p. 159).

This relates to the other criticism of ancient Egyptian mathematics, concerning the nature of proof or lack thereof. Gillings (1972/1982, pp. 145-146, 232-234) and Joseph (1991, pp. 82-83, 127-129) both treat this issue in some detail. Gillings's observation concerning the rigor of Egyptian proof is worth noting:

> Twentieth-century students of the history and philosophy of science, in considering the contributions of the ancient Egyptians, incline to the modern attitude that an argument or logical proof must be *symbolic* if it is to be regarded as rigorous, and that one or two specific examples using selected numbers cannot claim to be scientifically sound. But this is not true! A nonsymbolic argument or proof *can* be quite rigorous when given for a particular value of the variable; the conditions for rigor are that the particular value of the variable should be *typical*, and that further generalization to *any* value should be *immediate*. In any of the topics mentioned in this book where the scribes' treatment follows such lines, both these requirements are satisfied, so that the arguments adduced by the scribes are already *rigorous*; the concluding proofs are really not necessary, only confirmatory. The rigor is implicit in the method. (1972/1982, pp. 233-234)

For our students, some of whom have experienced repeated failure in their trajectory studying mathematics and, in the process, position themselves as well as are positioned as mathematically underprepared, this augmentation in their confidence and self-respect is nontrivial and, in some cases, leads to their willingness to pursue mathematics beyond mere institutional requirements.

What is an Ethnomathematical Perspective?

Ethnomathematics is a discipline that emerged from a politically engaged multicultural perspective on mathematics and mathematics education. Theorization about ethnomathematics was initiated and elaborated on by, mathematician and educational theorist, Ubiratan D'Ambrosio (1985; 1987; 1988; 1990; 2001). In Powell and Frankenstein (1997), we review the development of various definitions and associated perspectives on ethnomathematics (1997, pp. 5-11). For us, ethnomathematics attempts to correct the history of mathematics for African and other formerly colonized peoples disempowered by a varied, a violent, and an avaricious European colonial process and currently threatened and plummeted by a pernicious imperial project euphemistically called globalization, one ancillary tentacle of which is the current misdirected "war against terrorism." In this perspective, we also include concerns about the politics of knowledge and cultural underpinnings and interactions of mathematical ideas.

In his recent book, *Etnomatemática: Elo entre as Tradições e a Modernidade* [Ethnomathematics: Link between Tradition and Modernity], D'Ambrosio (2001) articulates his viewpoint on the political nature of the discipline:

> Ethnomathematics is the mathematics practiced by cultural groups such as urban and rural communities, labor groups, professional classes, children of a certain age bracket, indigenous societies, and many other groups that identify themselves through objects and traditions common to the groups.
>
> Beside this anthropological character, ethnomathematics has an indisputable political focus. Ethnomathematics is imbued with ethics, focused on the recuperation of cultural dignity of human beings.
>
> The dignity of the individual is violated by social exclusion that often causes one not to pass discriminatory barriers established by the dominate society, including and, principally, in schools (p. 9, our translation[17]).

In this viewpoint, a conceptually fruitful consequence of defining ethnomathematics as specific mathematical practices constituted by cultural groups is the scope of such activities. One can theorize that in ethnomathematics, the prefix "ethno" not only refers to a specific ethnic, national, or racial group, gender, or even professional group but also to a cultural group defined by a philosophical and ideological perspective. The social

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[17] Etnomatemática é a matemática praticada por grupos culturais, tais como comunidades urbanas e rurais, grupos de trabalhadores, classes profissionais, crianças de uma certa faixa etária, sociedades indígenas, e tantos outros grupos que se identificam por objetivos e tradições comuns aos grupos.
Além desse carácter antropológico, a etnomatemática tem um indiscutível foco político. A etnomatemática é embebida de ética, focalizada na recuperação da dignidade cultural do ser humano.
A dignidade do indivíduo é violentada pela exclusão social, que se dá muitas vezes por não passar pelas barreiras discriminatórias estabelecidas pela sociedade dominante, inclusive e, principalmente, no sistema escolar (p. 9).

and intellectual relations of individuals to nature or the world and to such mind-dependent, cultural objects as productive forces influence products of the mind that are labeled mathematical ideas. For example, Dirk J. Struik (1948 and reprinted in 1997), an eminent mathematician and historian of mathematics, indicates how a particular perspective—dialectical materialism—decisively influenced Marx's theoretical ideas on the foundation of the calculus. The calculus of Marx (1983) represents the ethnomathematical production of a specific cultural group. [18]

Another important consequence of D'Ambrosio's viewpoint is that by highlighting the culturally damaging consequences of social exclusion, ethnomathematics breaks with attributes of Enlightenment thinking. It departs from a binary mode of thought and a universal conception of mathematical knowledge that privileges European, male, heterosexual, racist, and capitalistic interests and values. D'Ambrosio (2001) puts it this way:

> Ethnomathematics encompasses this reflection on decolonialization and in the search for real possibilities of access for the subaltern, for the marginalized and for the excluded. The most promising strategy for education, in societies that are in transition from subordination to autonomy, is to reestablish the dignity of its individuals, recognizing and respecting their roots. To recognize and respect the roots of an individual

[18] For a popular account of Marx's calculus, see Gerdes (1985).

> does not mean to ignore and reject the roots of the other, but, in a process of synthesis, to reinforce ones own roots (p. 42, our translation[19]).

As D'Ambrosio (2001) notes, this research program contains other dimensions including conceptual, historical, cognitive, epistemological, and educational. As ethnomathematics educators, we are not neutral academics, but activist academics, committed to finding ways to contribute to struggles for justice through our educational work. We are not just interested, for example, in the mathematics of Angolan sand drawings, but also in the politics of imperialism that arrested the development of this cultural tradition, and in the politics of cultural imperialism that discounts the mathematical activity involved in creating Angolan sand drawings. Further, we are alert for ways that this contextualized mathematical knowledge can be used in educational settings to encourage greater justice in society culturally, socially, economically, and politically.

Conclusion: Respecting Intellectual Diversity

A cornerstone for a pedagogical approach that advances education for justice in the context of ethnomathematics involves actively respecting intellectual diversity. Accepting and respecting intellectual diversity leads us to reconsider all our knowledge of the world and to recognize that there is much about the world of which we have no knowledge. Pinxten, van Doren, and

[19] A etnomatemática se encaixa nessa reflexão sobre a descolonização e na procura de reais possibilidades de acesso para o subordinado, para a marginalizado e para o excluído. A estratégia mais promissora para a educação, nas sociedades que estão em transição da subordinação para a autonomia, é restaurar a dignidade de seus indivíduos, reconhecendo e respeitando suas raízes. Reconhecer e respeitar as raízes de um indivíduo não significa ignorar e rejeitar as raízes do outro, mas, num processo de síntese, reforçar suas próprias raízes. (p. 42)

Harvey (1983) argue that "[a]s long as science cannot pretend to have valid answers to all basic questions…it is foolish to exterminate all other, so-called primitive, pre-scientific or otherwise foreign approaches to world questions" (1983, p. 174).[20]

One of the most significant contributions of Paulo Freire (1982) to our thinking about what counts as knowledge is the idea that:

> Our task is not to teach students to think—they can already think, but to exchange our ways of thinking with each other and look together for better ways of approaching the decodification of an object.

This idea is critically important because it implies a fundamentally different set of assumptions about people, pedagogy, and knowledge-creation. Because some people in the United States, for example, need to learn to write in "standard" English, it does not follow that they cannot express in their own language very complex analyses of social, political, economic, ethical and other issues. And many people with an excellent grasp of reading, writing, and mathematics skills need to learn about the world, about philosophy, about psychology, about justice, and about many other areas in order to deepen their understanding of the world. Marcuse (1964) argues that in our society the rational, sophisticated calculations of, for example, nuclear kill and over-kill, are

[20] An example of a scholar who would be labeled "primitive" or "pre-scientific" is discussed by Brown (1997, p. 37): "The life of Ramanujan supports the view that innocence may be an asset in much of mathematical thinking. As a matter of fact, Ramanujan, who had received minimal formal mathematical training, came upon the most remarkable connections, and many of his arguments defied accepted cannons of proof. That many of his conclusion were wrong is beside the point, for given his untutored notion of proof and his lack of formal education, it is noteworthy that he was able to come upon so many discoveries and in fact to create so many new fields."

truly irrational, obscuring the only rational response to nuclear holocaust—resistance:

> [I]n this society, the rational rather than the irrational becomes the most effective mystification. (p. 189)
>
> For example, the scientific approach to the vexing problem of mutual annihilation—the mathematics and calculations of kill an over-kill, the measurement of spreading or not-quite-so-spreading fallout…is mystifying to the extent to which it promotes (and even demands) behavior which accepts the insanity. It thus counteracts a truly rational behavior—namely, the refusal to go along, and the effort to do away with the conditions which produce the insanity. (p. 190)

In a similar vein, D'Ambrosio (1997) argues:

> The mission of bringing Western civilization to the planet has been the essence of conquest and of the colonial enterprise. Now we are at a crossroads. The human species and the planet itself are threatened…
>
> The only possibility for survival depends on a better understanding of the entire set of possible explanations and views of the individual, of society, of nature, of the cosmos. Western mathematics, the most perfect embodiment of Western civilization, cannot be immune from the search for this deeper understanding. We can benefit much from understanding the workings of different systems of knowledge, the same way a stranger can tell us much about ourselves. (p. 15)

> There is no future in denying some successes in the science and technology developed following the Greek style. We will surely not be able to build faster jets and more powerful missiles using the male and female triangles of the Xingu ethnomathematics. But maybe the male and female triangles could help us *not* build the missiles and the jets carrying the bombs. (p. 17)

In a non-trivial way we can learn a great deal from intellectual diversity. Most of the burning social, political, economic and ethical questions of our time remain unanswered. In the United States we live in a society of enormous wealth and we have significant hunger and homelessness; although we have engaged in medical and scientific research for scores of years, we are not any closer to changing the prognosis for most cancers. Certainly we can learn from the perspectives and philosophies of people whose knowledge has developed in a variety of intellectual and experiential conditions. Currently "the intellectual activity of those without power is always labeled non-intellectual" (Freire & Macedo, 1987, p. 122). When we see this as a political situation, we can realize that all people have knowledge, all people are continually creating knowledge, doing intellectual work, and all of us have a lot to learn.

Brown’s attention to the emotional aspects of doing mathematics, the connection of mathematics to society and culture, the value implications of real-world problems, and studying incompatible perspectives on problems, all suggest a broader, political concept of humanistic mathematics education that we have developed here and in our other work. Respecting intellectual diversity is politically important precisely since the knowledge of non-Western and indigenous peoples has been violently devalued. In mathematics classrooms, the politics of knowledge needs to be part of the mathematical conversation and in this way we can develop respect for intellectual diversity.

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