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Bridging Past and Present:
Ethnomathematics, the *Ahmose Mathematical Papyrus*,
and Urban Students

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Abstract

This paper concerns a course module that incorporates scholarship based on mathematical insights documented by Ahmose, the ancient Egyptian scribe writing circa 1650 BC. It engages urban students in reflecting on their daily, informal experience with undoing physical and mental operations and relates these to algebraic techniques suggested in Problems 24 through 34 of the *Ahmose Mathematical Papyrus*. Students build their mathematical understanding so that they can own that knowledge and empower themselves *before* appreciating historical connections of the *Ahmose Mathematical Papyrus* to their work. Beyond the classes of equations prescribed in the curricula of middle and secondary schools, as well as developmental mathematics courses at universities, students generalize ancient techniques using circle equations to solve numerous classes of more complex algebraic equations such as n -degree equations, where n is rational and $n > 2$, radical equations expressed in exponential form, as well as logarithmic equations. Based on African algebraic techniques, students develop sophisticated mathematical insights and abilities and greater self-confidence as learners of mathematics. Since *all* students have a common biological heritage in Africa, they gain an increased appreciation for the mathematical accomplishments of their ancestors and for the diverse cultural manifestations of mathematical ideas. Moreover, students inquire into the politics of social structures that devalue the intellectual contributions of Africans and engage more deeply in academic mathematics. In conclusion, this paper poses questions for further inquiry into the mathematics of ancient Egypt.

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This paper outlines an ethnomathematics project, a course module, which combines historical scholarship and pedagogical innovation along with mathematical insights to engage urban students in the study of algebraic equations. The historical material incorporated into the module concerns ancient mathematical ideas inscribed on an Egyptian papyrus circa 1650 BC or, as we shall represent such dates, -1650 by the scribe Ahmose. The epistemological perspective and pedagogical tools of the module are based on the work of Hoffman and Powell (1988; 1991). The module engages students in learning algebraic techniques that reflect their daily, informal experience with undoing both physical and mental operations and relates these to algebraic ideas, techniques, and challenges suggested in Problems 28 to 34 of the *Ahmose Mathematical Papyrus*.¹ Some of the urban students with whom we work, considered by their respective academic institutions as mathematically underprepared, learn to solve not only single-variable, first- and second-degree equations, the only two classes of equations that at present are part of most algebra curricula, but also to solve with sophisticated facility n -degree equations (where $n > 2$) as well as radical equations (expressed in radical or exponential form). Importantly, far from trivializing or folklorizing Africa or its

* We thank Marilyn Frankenstein, University of Massachusetts, College of Public and Community Service, Boston, USA; and Mark Jacobs, Cape Technikon, Cape Town, South Africa, for their useful comments on an earlier draft.

¹ In the preface to their book, Chace, Manning, and Archibald (1929) indicate that in the original papyrus the problems are not numbered and credit Eisenlohr with numbering them.

mathematical contributions, the inclusion of African algebraic concepts help students develop more complex mathematical insights and abilities.

In this paper, we first briefly indicate the setting of our work and then present our course module and pedagogical approach. The presentation of the module will include its central multivalent tool²—circle equations—and indication of areas to which the tool can be applied that are beyond the scope our implementation. Then we discuss the historiography of the *Ahmoose Mathematical Papyrus* as well as indicate why, from pedagogical and political perspectives, we engage students in the mathematics before the historical and cultural ideas within which the mathematical exploration is embedded. In conclusion, we discuss the significance of the curricular module, raise some questions for further investigation, and suggest an agenda for the ethnomathematical examination of historical text and the creation of mathematical engagements for students in or outside of school classrooms in urban environments.

Setting

Our work is set in two urban areas in the United States: New York City in the State of New York and Newark in the State of New Jersey. One author (Temple) teaches mathematics in a middle school, De La Salle Academy, while the other author's (Powell) students are in their first year at Rutgers University. In both cases, our students are largely African American and Latino. Most of our students come from low-income, working-class families, living in difficult urban environments. By standard measures of academic achievement, the students who enroll in each of our institutions are within the top 10 to 15 percent of the student body from their previous schools. Nevertheless, in the context of each institution, our students are described as having academic promise though underprepared for the rigors of the institution's mathematics curriculum.

Specifically, the students at the university have had instruction in algebra in secondary school but because of their low achievement in the subject are required to take a developmental course in it. While a goal of our institutions is to improve students mathematical abilities, their previous engagement with mathematics and their positioning within the institution's curriculum have constituted them as low achievers and consequently students have low self-confidence in themselves as mathematics learners.

Such students in many schools throughout the United States are regularly and institutionally discouraged from viewing themselves as having the potential to pursue mathematics beyond computation. Urban, working-class students in the 5th and 6th grades (10- to 12-year olds) are more often than not tracked away from studying algebra in the 7th grade. As a consequence, in high school, they attend courses that do not prepare them adequately for university-level mathematics. Further, these students are disproportionately African American and Latino. Our course module focuses on enabling such students to engage in activities that will reveal to them that they have the intellectual capacity to pursue mathematics at high levels. In the case that students are in developmental courses in their first year of university, the course module is designed to provide them with insights into their capability to pursue courses of study that include mathematics and mathematics-related disciplines.

The Course Module: Circle Equations

The importance of Africa's contribution to mathematics and the central role of that contribution to the mathematics studied in schools have not received the attention and understanding that befit them³. Important projects to redress

² As Hoffman and Powell (1991) define it, a "multivalent pedagogical tool", [is] a technique and variations of it, which can be used as a vehicle for teaching and learning a number of interconnected topics" (p 91).

³ Diop (1974) argues a more general point about historiography: "The history of Black Africa will remain suspended in air and cannot be written correctly until African historians dare to connect it with the history of Egypt....The ancient Egyptians were Negroes. The moral fruit of their

this state of affairs have taken root among some historians (Diop, 1974; Jackson, 1970), historians of mathematics (Gillings, 1982; Katz, 1998) as well as by scholars of ethnomathematics (Gerdes, 1989, 1992, 1999; Joseph, 1991; Lumpkin, 1985, 2002; Lumpkin & Powell, 1995; Zaslavsky, 1999). Documentary evidence of insightful and critical algebraic ideas developed in ancient Egypt exists, but little of this information is taught to students studying mathematics, at any level. The course module we have implemented contributes to redressing this lack of scholarship in school algebra courses by incorporating mathematical ideas from what some researchers correctly call the *Ahmoose Mathematical Papyrus* (Diop, 1985; Joseph, 1991; Lumpkin, 1985).

This ancient Egyptian mathematical text, often known as the Rhind Mathematical Papyrus, now housed in the British Museum (except for a few fragments in the Brooklyn Museum), is the inspiration for the course module. The module builds on three mathematical ideas present in algebraic equations extant in Egypt during the time Ahmoose wrote his mathematical papyrus (circa – 1650): (a) the concept of unknown or variable quantities (Boyer & Merzbach, 1989, pp. 15-16; Gillings, 1982, p. 154); (b) undoing or, equivalently, inverse operations (Katz, 1998, p. 15); (c) and “think of a number” problems (Gillings, 1982, p. 181-184). To engage students in developing an awareness⁴ of the role

civilization is to be counted among the assets of the Black world....that Black world is the very initiator of the ‘western’ civilization flaunted before our eyes today. Pythagorean mathematics...and modern science are rooted in Egyptian cosmogony and science” (p. xiv).

⁴In this paper, we use the term ‘awareness’ or ‘mathematical awareness’ in the technical sense suggested by Gattegno (1987) and elaborated on by Powell (1993, p. 358):

Gattegno makes clear that, for learners, learning or generating knowledge occurs not as a teacher narrates information but rather as learners employ their will to focus their attention to educate their awareness. Learners educate their awareness as they observe what transpires in a situation, as they attend to the content of their experiences. As a learner remains in contact with a transpiring experience, awareness proceeds from “a dialogue of one’s mind with one’s self” about the content of that which one experiences (Gattegno, 1987, p. 6). One’s will, a part of the active self, commits one to focus one’s attention so that the mind observes the content of one’s experience and, through dialogue with the self, becomes aware of particularities of one’s experience. Specifically, in mathematics, the content of experiences, whether internal or external to the self, *can be*

undoing or inverse operations can play in solving certain types of equations, one can present a type of “think of a number problem” as appears in Figure 1:

“I’m thinking of a number. I subtract 11 from it, multiply by 3, and add 2. The result is 80. What’s my number?”

Figure 1. A “think of a number” problem.

This is an example of a “What’s my number?”⁵ problem that is likely to trigger in students an awareness of undoing that includes the ideas of using inverses: multiplicative and additive inverses or inverse operations as well as the reversal of the order of operations. Undoing is a mathematical process related to many aspects of daily life: dressing and undressing, wrapping and unwrapping parcels, and so forth. In schools and textbooks, this quotidian process is often not related to mathematics but, of course, is fundamental to it. Fortunately, ethnomathematics resurrects links between quotidian and formal knowledge schemas.

Students are asked to reflect on their undoing technique so as to specify completely its constituent processes and ideas. After students discuss their ideas, we introduce conventional terminology for labeling processes and ideas.⁶ For the “think of a number” problem in Figure 1, students eventually might describe their process for solving it as follows:

The first operation or action performed on the original number thought of was to “subtract 11.” The second action was to “multiply by 3.” The third and last action was to “add 2,” and the result was “80.” To undo the problem, we need to start at the end. Immediately before the result 80

feelings, objects, relations among objects, and dynamics linking different relations [italics added] (Gattegno, 1987, p. 14).

⁵ We use interchangeably the phrases “think of a number” and “What’s my number?” problems.

⁶ This pedagogical approach is akin to the “five crucial steps in the Algebra Project curriculum process” (Moses & Cobb, 2001, p. 120). In both approaches, one begins with the experiential, mathematical world of learners and assists them to connect their ideas to the symbolic world of formal, academic mathematics.

was obtained, the last action performed was to add 2. To undo *add 2* we can *subtract 2* from 80, which is 78. Now, the action performed before 78 was obtained was to multiply by 3. To undo *multiply by 3* we can *divide by 3*. So, we take 78 and divide it by 3, which gives 26. The action performed before 26 was obtained was to subtract 11. To undo *subtract 11* we can *add 11*. So, the original number thought of was 37.

The above description reveals an awareness of the mathematical ideas of inverse operations as well as reverse order of operations as useful for solving the “think of a number” problem given in Figure 1. This awareness is generalizable, and students use it to handle more complex “think of a number” problems such as the one in Figure 2:

“I’m thinking of a number. I raise it to the third power, add 8, raise it to the one-half power, add 1, and multiply it by 6. My result is 30. What’s my number?”

Figure 2. A “think of a number” problem involving integral and fractional exponents.

As the number and the complexity of the operations increase, students tend to develop *ad hoc* notational devices or inscriptions to represent “think of a number” problems. One notational device that we offer is circle equations (Hoffman & Powell, 1988, 1991). The circle equation for the problem in Figure 2 is given in Figure 3.

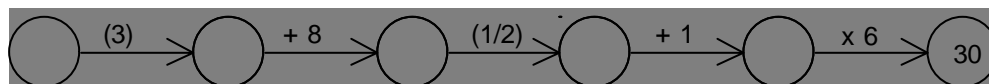


Figure 3: The circle equation of the “think of a number” problem given in Figure 2.

In circle equations there are several conventions. The first circle represents the number referred to by the phrase “I’m thinking of a number.” The expressions above the right-facing arrows correspond to the operators in the

“think of a number” problem in the order in which they are mentioned. Finally, numbers placed inside parentheses are considered to be exponents.

Besides being a tool to represent graphically “think of a number” problems, circle equations can also be used, with slight modifications, to depict a process of solving such problems. Explicit in the structure of circle equations, read from left to right, is the order in which operations are performed in “think of a number” problems. Similarly, but reading from right to left, the process of undoing or solving can also be represented. Figure 4 contains a “think of a number” problem, its translation into a circle equation, and indication of how solving it can be symbolized, using left-facing arrows with inverse operations written below them.

“I’m thinking of a number. I take its un-factorial, subtract 1, raise to the third power, and divide by 5. The result is 25. What’s my number?”

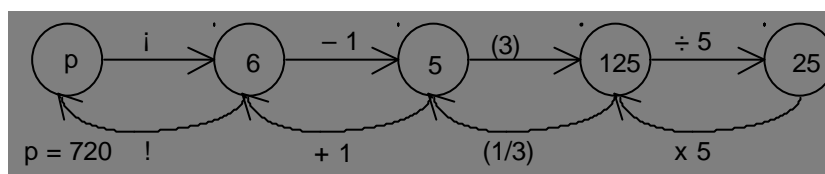


Figure 4. A “think of a number” problem and its corresponding circle equation with its solution indicated.

In the statement of the “think of a number” problem in Figure 4, the first operation is “un-factorial,” which is a non-conventional operation, suggested by a student, that is the inverse of factorial, and whose notational sign, also suggested by a student, is the 180°-rotational inverse of !, the notation for factorial. Moreover, in the circle equation, the circle that stands for the phrase, “I’m thinking of number,” contains a variable, which means that the circle equation can be translated into the standard notation of academic mathematics (see Figure 5).

$$(a) 3(x-11)+2=80; \quad (b) 6\left\{(y^3+8)^{\frac{1}{2}}+1\right\}=30; \quad (c) \frac{(p-1)^3}{5}=25$$

Figure 5. The standard notation for the “think of a number” problems given in Figures 1, 3, and 4, respectively, (a), (b), and (c).

Each of the equations in Figure 6, written in standard form, can be read as a “think of a number” problem or written in circle notation. Students develop facility solving equations by reading standard or circle equations as “think of a number” problems. As Hoffman and Powell note, they

have three alternative representations of an equation: the *verbal* ‘What’s my number?’ statement of the problem, the *graphical* circle equation, and the *symbolic* standard notation. These representations provide pupils with bases for forming both verbal and visual images of equations. On paper or in the mind, [students] can translate one representation into the other two. Consequently, these three equivalent representations of equations provide [students] with a richer set of connections and platforms from which to construct meanings than conventional introductions to solving equations (p. 95).

Eventually, in their minds, they translate among standard and circle equations, and “think of a number” problems.⁷

$$\mathbf{Erro!} - 2 = 1; \quad 5 \left(\sqrt{\frac{((\log_5 y) - 1)^7}{32}} \right) = 10$$

Figure 6. Two complex looking equations whose solution can be found using Ahmose’s mathematical insights.

By working among these representations, students learn that equations such as the ones in Figure 6 simply appear complicated but are conceptually no more difficult to solve than an equation such as $3(x + 6) = 24$. The difficulty of the equations in Figure 6 arises from the amount of time required to solve them

⁷ On the same point, Hoffman and Powell (1991) indicate that “with practice, pupils can imagine that a given equation is expressed as a circle equation and solve the equation, as it were, by sight”

rather than any intrinsic complexity that they possess. Students learn that equations such as the ones in Figure 6 are, in fact, not difficult but simply laborious. Conjoined with this awareness is an increase in their mathematical self-confidence. For they are able to solve equations that more advanced mathematics students initially find baffling. For our students, some of whom have experienced repeated failure in their trajectory studying mathematics and, in the process, position themselves as well as are positioned as mathematically underprepared, this augmentation in their confidence is nontrivial and, in some cases, leads to their willingness to pursue mathematics beyond mere institutional requirements.

Ahmose Mathematical Papyrus: Historical Inquiry

Ethnomathematical perspective

Besides understanding academic mathematics, which, as Moses and Cobb (2001) argue, in today's society is essential for engaged citizenship, students must also have a critical and analytic understanding of history. History is important to the identity of students. However, memorized history without understanding is simply information, comparable to resources existing without having access to them. For essentially two reasons, we challenge our students to build their mathematical understanding so that they can own that knowledge and empower themselves *before* appreciating historical connections to their work. First, students enter class with the expectation that they will do mathematics; this is the social contract of schooling. Second, because of this expectation, to ask students to appreciate historical facts related to mathematics before doing mathematics risks sending a message to students that they are incapable of doing mathematics. The mathematical understanding that students develop is a bridge that traverses nearly 4000 years to connect them to the *Ahmose Mathematical*

(p. 95). In this sense, then, circle equations assist learners to develop dynamic imagery of equations that allows them to solve equations without pencil and paper.

Papyrus (AMP) that inspires our course module. Students study the history of the AMP as well as differing scholarly interpretations of the contributions of ancient Egypt to world mathematics.

Understanding what mathematics ancient Egypt contributed and its value are problematic issues. Many historians have narrated historical accounts of ancient Egyptian mathematics with unabashed cultural bias, and, as such, discussing the contributions of ancient Egyptians to world mathematics poses a challenge. Insights into the challenge can be acquired by observing what some notable historians and mathematicians have had to say about ancient Egyptian mathematics:

The table [of the AMP Recto]⁸ is in itself a monument to the lack of the scientific attitude of mind in the Egyptians. (Peet, 1931, as quoted in Gillings, 1972/1982, p. 232)⁹

The Greeks may also have taken from the Egyptians the rules for the determination of areas and volumes. But for the Greeks, such rules did not constitute mathematics; they merely led them to ask; how does one prove this? (Van Der Waerden, 1954, as quoted in Gillings, 1972/1982, p. 232)¹⁰

⁸ The Recto in the AMP is a reference table of decompositions of $2/n$ into unit fractions, where n is an odd number from 3 through 101. The table occupies approximately the first one-third of the papyrus scroll. For a discussion of the possible mathematical criteria used by Ahmose to construct the Recto, see Gillings (1972/1982, p. 45-80; 1974).

⁹ This supposed “lack of scientific attitude of mind” reveals interesting mathematical properties and consequences as well as was incorporated into the work of Greek mathematicians. See Gillings (1972/1982, pp. 45-80).

¹⁰ For a discussion of the nature of proof in Egyptian mathematics, see Gillings (1972/1982, pp. 145-146, 232-234) and Joseph (1991, pp. 82-83, 127-129). Gillings’s observation concerning the rigor of Egyptian proof is worth noting:

Twentieth-century students of the history and philosophy of science, in considering the contributions of the ancient Egyptians, incline to the modern attitude that an argument or logical proof must be *symbolic* if it is to be regarded as rigorous, and that one or two specific examples using selected numbers cannot claim to be scientifically sound. But this is not true! A nonsymbolic argument or proof *can* be quite rigorous when given for a particular value of the variable; the conditions for rigor are that the particular value of the variable should be *typical*, and that further generalization to *any* value should be

When one compares Egyptian and Babylonian accomplishments in mathematics with those of earlier and contemporary civilizations, one can indeed find reason to praise their achievements. But judged by other standards, Egyptian and Babylonian contributions to mathematics were almost insignificant, although these same civilizations reached relatively high levels in religion, art, architecture, metallurgy, chemistry, and astronomy. Compared with the accomplishments of their immediate successors, the Greeks, the mathematics of the Egyptians and Babylonians is the scrawling of children just learning how to write as opposed to great literature. (Kline, 1962, p. 14)

These citations demonstrate intellectual, cultural bias pervasive in many historical assessments of ancient Egyptian achievements in mathematics. In contrast, Newman (1956) presents a more culturally sensitive approach toward understanding Egyptian mathematics:

It seems to me that a sound appraisal of Egyptian mathematics depends upon a much broader and deeper understanding of human culture than either Egyptologist or historians of science are wont to recognize. As to the question how Egyptian mathematics compares with Babylonian or Mesopotamian or Greek mathematics, the answer is comparatively easy and comparatively unimportant. What is more to the point is to understand why the Egyptians produced their particular kind of mathematics, to what extent it offers a culture clue, how it can be related to their social and political institutions, to their religious beliefs, their economic

immediate. In any of the topics mentioned in this book where the scribes' treatment follows such lines, both these requirements are satisfied, so that the arguments adduced by the scribes are already *rigorous*, the concluding proofs are really not necessary, only confirmatory. The rigor is implicit in the method. (1972/1982, pp. 233-234)

practices, their habits of daily living. It is only in these terms that their mathematics can be judged fairly. (p. 178)

Interestingly, Newman here sounds like an early ethnomathematician, commenting about three decades before ethnomathematics emerged as a research program within an international community. We agree with him that a worthwhile inquiry into Egyptian mathematics must be conducted within a cultural perspective, understanding it from, to use an ethnographic notion, an emic perspective.¹¹ Undoubtedly, we cannot avoid understanding Egyptian mathematics in light of our own culture's mathematics. However, despite the tendency to compare cultural achievements, which can lead to unfair evaluations, this does not preclude the validity of attempting to view Egypt from within Egypt.

The problem lies in interpreting the significance of Egyptian mathematics through the lens of Egyptian culture, which is a challenge that ethnomathematics addresses. Moreover, ethnomathematics also inquires into the politics of knowledge that discounts various mathematical activities, denigrating the ideas of certain cultural groups as less abstract or less sophisticated than a supposed normative cultural group. In a recent book, *Etnomatemática: Elo entre as tradições e a modernidade* [Ethnomathematics: Link between tradition and modernity], D'Ambrosio (2001) articulates a view of ethnomathematics that attends to its political nature:

Ethnomathematics is the mathematics practiced by cultural groups such as urban and rural communities, labor groups, professional classes, children of a certain age bracket, indigenous societies, and many other

¹¹ Kline would have done well to follow the methodological hint of Newman, who published his four-volume treatise eight years before Kline published his "cultural" approach to the history of mathematics.

groups that identify themselves through objects and traditions common to the groups.¹²

Beside this anthropological character, ethnomathematics has an indisputable political focus. Ethnomathematics is imbued with ethics, focused on the recuperation of cultural dignity of human beings.

The dignity of the individual is violated by social exclusion that often causes one not to pass discriminatory barriers established by the dominant society, including and, principally, in schools (p. 9, our translation¹³).

By highlighting the damaging consequences of social exclusion, ethnomathematics represents a break with attributes of Enlightenment thinking. It departs from a binary mode of thought and a universal conception of mathematical knowledge that privileges European, male, heterosexist, racist, and capitalistic interests and values.

Similarly, our view is to avoid privileging and instead to reckon with the AMP itself. It is our principal and widest window into the mathematical thinking of ancient Egyptians. As a text, since Ahmose did not state the intended

¹² Defining ethnomathematics as specific mathematical practices constituted by cultural groups is conceptually fruitful since it broadens the scope of practices considered to be mathematical. One can theorize that in ethnomathematics, the prefix “ethno” not only refers to a specific ethnic, national, or racial group, gender, or even professional group but also to a cultural group defined by a philosophical and ideological perspective. The social and intellectual relations of individuals to nature or the world and to forces of production influence products of the mind that are labeled mathematical ideas. Dirk J. Struik (1948 reprinted in 1997), an eminent mathematician and historian of mathematics, indicates how a particular perspective—dialectical materialism—decisively influenced Marx’s theoretical ideas on the foundation of the calculus. The calculus of Marx (1983) represents the ethnomathematical production of a specific cultural group.

For a popular account of Marx’s calculus, see Gerdes (1985).

¹³ Etnomatemática é a matemática praticada por grupos culturais, tais como comunidades urbanas e rurais, grupos de trabalhadores, classes profissionais, crianças de uma certa faixa etária, sociedades indígenas, e tantos outros grupos que se identificam por objetivos e tradições comuns aos grupos.

Além desse carácter antropológico, a etnomatemática tem um indiscutível foco político. A etnomatemática é embebida de ética, focalizada na recuperação da dignidade cultural do ser humano.

audience of his mathematical narrative, there is no basis to judge or compare the relative standing of his work. Historians and other scholars have no idea whether the AMP was a great work or a minor one, a compendium for scholars or a manual for clerks or schoolchildren (Newman, 1956, p. 170). Comparative commentary on the quality and sophistication of the AMP by a historian, mathematician, or pedagogue most probably reflects the bias, ignorance, or interpretative injudiciousness of the commentator.¹⁴

One question that history often omits and that warrants mentioning, as Diop (1974) does, is “What were the Egyptians?” He writes that “eyewitnesses of that period formally affirm that the Egyptians were Blacks” (p. 1). The “black” complexion of the ancient Egyptians is left unattended in most historical descriptions of their culture, science, mathematics, and technology. The blackness of the Egyptians can help to explain the dismissive framing of Egyptian mathematics and, by comparison, the glorification of Greek mathematics. Undoubtedly, some Eurocentrism accounts for the differential historiography of Greek and Egyptian mathematics, especially for the claim as to whose mathematics first exhibited abstraction and intellectual prowess. We hope to contribute to the recognition and pedagogical use of Egyptian mathematics, while cognizant that truth about it is hard to obtain given traditions of biased research on ancient Egypt, and the incomplete glimpse we have of ancient Egypt based on extant constructions, papyri, carvings, and other material products from its antiquity.

One honest historian was the scribe Ahmose himself, who announces to his readers that the papyrus that he penned, in red and black ink, around –1650

A dignidade do indivíduo é violentada pela exclusão social, que se dá muitas vezes por não passar pelas barreiras discriminatórias estabelecidas pela sociedade dominante, inclusive e, principalmente, no sistema escolar. (D’Ambrosio, 2001, p. 9)

¹⁴ Even nowadays, some scholars make claims about the AMP without providing evidence. For instance, Dudley (2002) claims that the “Rhind papyrus” (that is, the AMP) is “the world’s first mathematics textbook that we know about” and that “the problems were, as could be expected,

is a copy of a document that dates back some 200 years earlier. Ahmose's papyrus scroll, humanity's oldest extant, written mathematical document, measures about 6 meters long and one-third of a meter wide. According to Robins and Shute (1987), the Ahmose Mathematical Papyrus originally formed a continuous roll of fourteen sheets of papyrus, each sheet measuring approximately 40 cm wide by 32 cm high. To form the scroll, the sheets were gummed together at their edges. Written in hieratic form,¹⁵ Ahmose's papyrus contains numerical tables for calculations as well as arithmetic, geometric, and algebraic problems, 87 in all, together with some explanations of methods and worked out solutions (Chace et al., 1929; Gillings, 1982).

Although the ravages of more than three millennia of time, conquest, plunder, and colonialism have destroyed many African documents and other artifacts of historical and scientific significance, a few Egyptian papyri have managed to survive. Ahmose's papyrus scroll was found in the ruins of a small building close to the mortuary temple of Ramesses II at Thebes and, in 1858, purchased in Luxor along with other Egyptian antiquities by Alexander Henry Rhind. After his death in 1963, the papyrus scroll, "together with another mathematical document known as the Leather Roll (BM 10250), was purchased from his executor in two pieces (BM 10057-58) by the British Museum in 1865" (Robins & Shute, 1987, p. 9). Subsequently, the papyrus scroll was named after him as the Rhind Mathematical Papyrus. Nevertheless, following the example of Boyer and Merzbach (1989) and Joseph (1991), we choose to refer to the papyrus as the *Ahmose Mathematical Papyrus* (AMP), since Ahmose is known and is the contributor of this ancient Egyptian cultural product.

As we mentioned earlier, from a mathematical perspective, three ideas from the AMP inspired the course module involving circle equations. In the

primitive...it is no surprise that what they [the Egyptians] did was, to us, crude and limited" (p. 8).

¹⁵ Hieratic writing in ancient Egypt was a cursive form of their hieroglyphics, a writing system that used symbols or pictures to denote objects, concepts, or sounds.

following portion of this paper, we indicate where these ideas—variable, “think of a number” problem, and undoing—appear in the *Ahmoose Mathematical Papyrus*.

The mathematical variable

In the AMP, Ahmoose presents a series of problems dealing with methods of solving equations of the first degree. Of the 87 problems in the AMP, these occur among Problems 24 through 34. The mathematical concept of variable¹⁶ is illustrated in Problems 24 to 34. In Problems 24 to 27, the method of solving is false position (Boyer & Merzbach, 1989; Gillings, 1982). For Problems 30 through 34, the solution method involves division, using what in modern terms is called the multiplicative property of equality. In some of these problems Ahmoose uses factoring. Boyer and Merzbach (1989) state that factoring is used in only Problem 30, which is actually Problem 33, however, from our reading of the AMP plates and the free translation in Chace, Manning, and Archibald (1927, Plates 52 to 56), we claim that factoring and division is used not only in Problem 33 but also in Problems 31, 32 and 34. Importantly, in problems 24 to 27 and 30 to 34, the hieratic symbol for “aha” or “heap” appears and represents an unknown quantity whose value is to be determined. Although this symbol does not appear in Problems 28 and 29, these two problems also require one to determine the value of an unknown quantity. The absence of this hieratic symbol indicates that the two problems are of a different nature from Problems 24 to 27 and 30 to 34 in that they are similar to other classical problems called “think of number” problems

“Think of a number” problems

Challenges, puzzles, and games that express mathematical ideas are an omnipresent aspect of cultures (Bishop, 1988a, 1988b). The AMP also reveals a

cultural interest in challenges among ancient Egyptians. Problems 28 and 29, according to Gillings (1961; 1982) are “think of a number” problems (1982, p. 158) about which he considers them to be the earliest examples on record (1982, p. 181). Problem 28 is given in Figure 7, first with Chace, Manning, and Archibald’s translation and followed by a modern interpretation of it as given in Gillings (1982, p. 182).

Two thirds is to be added. One-third is to be subtracted.

There remains 10.

Make $\overline{10}$ [one-tenth]¹⁷ of this, there becomes 1. The remainder is 9.

$\overline{\overline{3}}$ of it namely 6 is to be added. The total is 15.

$\overline{\overline{3}}$ of this is 5. Lo! 5 is that which goes out, and the remainder is 10.

The doing as it occurs!

Let us state this in modern terms, adding a few clarifying details:

Think of a number, and add to it its $\overline{\overline{3}}$. From this sum take away its $\overline{\overline{3}}$ [one-thirds], and say what your answer is. Suppose the answer were 10. Then take away $\overline{10}$ of this 10, giving 9. Then this was the number first thought of.

Proof. If the number were 9, its $\overline{\overline{3}}$ is 6, which added makes 15. Then $\overline{\overline{3}}$ of 15 is 5, which on subtraction leaves 10. That is how you do it!

Figure 7. Problem 28 of the Ahmose Mathematical Papyrus.

In the AMP, Problem 28 is an example, among Problems 24 to 34, in which no aspect of it deals with practical quotidian concerns, geometric figures,

¹⁶ Naturally, a mathematical variable is a placeholder such as a letter, word, or phrase that represents a number or numbers as in $3y+1=5$ or $height(time)=time^2+2time+3$.

¹⁷ Gillings (1972/1982) represents unit fractions with a horizontal bar above the denominator and the fraction $2/3$ with two horizontal bars over 3. This is an adaptation from the ancient Egyptian hieratic and hieroglyphic forms of writing fractions in which $2/3$ is the sole exception to exclusive use of unit fractions.

arithmetic or geometric progressions, or computational tables and algorithms. In this sense, Problem 28 is representative of challenge problems in the AMP.

Undoing

Problems 30 to 34 are alike in the method of solution that Ahmose uses. All the problems involve the idea of division as the inverse of multiplication. For example, in Figure 8 we have Problem 33:

A quantity, its $\frac{2}{3}$, its $\frac{1}{2}$, and its $\frac{1}{7}$ added becomes 37. What is the quantity?
(Gillings, 1982, p. 159)

Figure 8. Problem 33 of the Ahmose Mathematical Papyrus.

This problem, $x + \frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x = 37$, here represented in modern notation as Boyer & Merzbach (1989) indicate was “solved by factoring the left-hand side of the equation and dividing 37 by $1 + \frac{2}{3} + \frac{1}{2} + \frac{1}{7}$, the result being $16 + \frac{1}{56} + \frac{1}{679} + \frac{1}{776}$ ” (p. 16). The last step of the solution process employs the idea of an inverse operation.

Significance of curricular module and questions for further research

This curricular module that incorporates scholarship on Africa into the curriculum of school algebra has manifold significance. First, students learn algebraic techniques from ancient Egypt that will allow them to solve first- and second-degree equations in a single variable. Beyond this, students learn to generalize these techniques to enable them to solve a wider array of and more complex algebraic equations. Second, students study the history of the *Ahmose Mathematical Papyrus* as well as the differing scholarly interpretations of the meaning of Problems 28 and 29. Third, since *all* students have a common biological heritage rooted in Africa (Diop, 1974; Tattersall, 1997), they gain an increased appreciation for the mathematical accomplishments of their ancestors as well as for the diverse cultural manifestations of mathematical ideas (Gerdes, 1999; Powell & Frankenstein, 1997).

From our work, a number of questions come to mind of which some are the following: What other areas of mathematics can be directly related to the AMP and other mathematical papyri of ancient Egypt? For any such area: What mathematical ideas and insights do they provide? How do these mathematical ideas relate to quotidian experiences? What issues about the politics of knowledge and mathematics education do these ideas raise? How might the incorporation of these ideas into school curricula be framed to expose racist ideology and challenge racism in the historiography of mathematics and mathematics education? How might a curricular module based on these ideas unveil the appropriation of non-Western scientific traditions without due credit?

We hope that this work contributes to efforts to incorporate ethnomathematical scholarship in schools so as to engage urban students more deeply in academic mathematics, to stimulate them to interrogate the politics of knowledge, and to encourage that they construct spaces for themselves and others to see engagement in mathematics as a right of being human.

References

- Bishop, A. J. (1988a). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht: Kluwer Academic Publishers.
- Bishop, A. J. (1988b). Mathematics education in its cultural context. *Educational Studies in Mathematics*, 19, 179-191.
- Boyer, C. B., & Merzbach, U. C., rev. editor. (1989). *A history of mathematics* (2nd ed.). New York: John Wiley & Sons.
- Chace, A. B., Manning, H. P., & Archibald, R. C. (1927). *The Rhind mathematical papyrus British Museum 10057 and 10058* (Vol. I). Oberlin, Ohio: Mathematical Association of America.
- Chace, A. B., Manning, H. P., & Archibald, R. C. (1929). *The Rhind mathematical papyrus British Museum 10057 and 10058* (Vol. II). Oberlin, Ohio: Mathematical Association of America.
- D'Ambrosio, U. (2001). *Etnomatemática: Elo entre as tradições e a modernidade [Ethnomathematics: Link between tradition and modernity]*. Belo Horizonte, MG: Autêntica.
- Diop, C. A. (1974). *The African origin of civilization: Myth or reality* (M. Cook, Trans.). Chicago: Lawrence Hill.
- Diop, C. A. (1985). Africa's contribution to world civilization: The exact sciences. In I. V. Sertima (Ed.), *Nile Valley Civilizations* (pp. 69-83). Atlanta, Georgia: Journal of African Civilizations.
- Dudley, U. (2002). The world's first mathematics textbook. *Focus: The Newsletter of the Mathematical Association of America*, 22(4), 8-11.
- Gattegno, C. (1987). *The science of education: Part 1: Theoretical considerations*. New York: Educational Solutions.
- Gerdes, P. (1989). Desenhos tradicionais na areia em Angola e seus possíveis usos na aula de matemática. *BOLEMA Especial*(1), 51-77.
- Gerdes, P. (1992). *Sobre o despertar do pensamento geométrica*. Curitiba: Universidade Federal de Paraná.
- Gerdes, P. (1999). *Geometry from Africa: Mathematical and educational explorations*. Washington, DC: The Mathematical Association of America.
- Gillings, R. J. (1961). Think of a number: Problems 28, 29 of the RMP. *Mathematics Teacher*, 54(2), 97-100.
- Gillings, R. J. (1972/1982). *Mathematics in the time of the pharaohs*. Mineola, New York: Dover.
- Gillings, R. J. (1974). The recto of the Rhind mathematical papyrus: How did the ancient Egyptian scribe prepare it? *Archive for History of Exact Sciences*, 12, 291-298.
- Gillings, R. J. (1982). *Mathematics in the time of the pharaohs*. Mineola, New York: Dover.
- Hoffman, M. R., & Powell, A. B. (1988). A multivalent tool for computation. *Mathematics in College*(Fall/Winter), 43-51.

- Hoffman, M. R., & Powell, A. B. (1991). Circle expressions and equations: Multivalent pedagogical tools. In D. Pimm & E. Love (Eds.), *Teaching and learning school mathematics* (pp. 91-96). London: Hadder & Stoughton.
- Jackson, J. G. (1970). *Introduction to African civilizations*. New York: Citadel.
- Joseph, G. G. (1991). *The crest of the peacock: Non-European roots of mathematics*. London: I. B. Tauris.
- Katz, V. J. (1998). *A history of mathematics: An introduction* (2nd ed.). Reading, MA: Addison-Wesley.
- Kline, M. (1962). *Mathematics: A cultural approach*. Reading, MA: Addison-Wesley.
- Lumpkin, B. (1985). Mathematics and engineering in the Nile Valley. In I. V. Sertima (Ed.), *Nile Valley Civilizations* (pp. 102-119). Atlanta, Georgia: Journal of African Civilizations.
- Lumpkin, B. (2002). Mathematics used in Egyptian construction and bookkeeping. *The Mathematical Intelligencer*, 24(2), 20-25.
- Lumpkin, B., & Powell, A. B. (1995). *Math: A rich Heritage*. Upper Saddle, NJ: Globe Fearon.
- Moses, R. P., & Cobb, C. E., Jr. (2001). *Radical equations: Math literacy and civil rights*. Boston: Beacon.
- Newman, J. R. (1956). *The world of mathematics* (Vol. 1). New York: Simon and Schuster.
- Powell, A. B. (1993). Pedagogy as ideology: Using Gattegno to explore functions with graphing calculator and transactional writing. In C. Julie & D. Angelis & Z. Davis (Eds.), *Proceeding of the second international conference on the political dimensions of mathematics education* (pp. 356-369). Cape Town: Maskew Miller Longman.
- Powell, A. B., & Frankenstein, M. (1997). *Ethnomathematics: Challenging eurocentrism in mathematics education*. Albany: State University of New York.
- Robins, G., & Shute, C. (1987). *The Rhind mathematical papyrus: An ancient Egyptian text*. New York: Dover.
- Tattersall, I. (1997). Out of Africa Again... And Again? *Scientific American*, 276(4), 60-67.
- Zaslavsky, C. (1999). *Africa counts: Number and patterns in African cultures* (3rd ed.). Chicago: Lawrence Hill.

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