During the past few decades several problems in mathematics have been solved by using techniques from classical and quantum field theories.

In this talk we will explore two problems in geometry and topology: the third Lie theorem for Lie algebroids and the deformation quantization of Poisson manifolds. We will see how a two dimensional string theory known as the Poisson sigma model (PSM) provides answers to these two problems. We will discuss recent generalizations of this construction and also introduce some open questions and ongoing work in this field.