The work of Galois in the early 19th century showed that the arithmetic of the rational numbers is reflected in a mysterious set of symmetries, known as the Galois group of \( \mathbb{Q} \). In the late 19th century, Felix Klein in his Erlangen program introduced the notion of symmetry into geometry. This "geometric" symmetry arises in the form of Lie-groups, seemingly quite different in nature from the Galois groups. In the 1960’s, Robert Langlands developed a tantalizing web of conjectures relating the two notions of symmetry. He postulated that the Galois group of \( \mathbb{Q} \) bears a close and specific relation to certain Lie groups well known in geometry, including the linear, symplectic, and orthogonal groups. An example of the fruitful and surprising results arising from such a relationship would be that the different algebraic extensions of \( \mathbb{Q} \) of degree \( n \) are governed by infinite-dimensional representations of the general linear group \( \text{GL}_n \).

I will introduce some of the ideas surrounding Langlands’ conjectures and then focus on more recent developments involving the so called local side of these conjectures, in which the rational numbers are replaced by the \( p \)-adic numbers.