Transfer and descent: some recent results

D. SHELSTAD[1]

Rutgers University

A basic tool for studying the transfer of representations is the dual transfer of orbital integrals. Here we report on some recent results for orbital integrals and in particular on a descent theorem [**LS3**].

Throughout, F is a local field of characteristic zero with algebraic closure $\overline{F}$ and $\Gamma = \mathrm{Gal}(\overline{F}/F)$; G is a connected reductive group defined over F.

§1 AN EXAMPLE

The notions of endoscopy and transfer require some care in formulation, but there is one example where we can give definitions quickly. Namely, suppose that H is the (unique up to F-isomorphism) quasi-split inner form of G. Then H is endoscopic for G in the sense of standard endoscopy (where no twisting by an automorphism is specified; see [**LS1**], [**KS**]). Up to isomorphism, it is the unique endoscopic group with dimension as great as that of G.

Fix an inner twist $\psi : G \to H$. Although ψ is not defined over F (unless G is quasisplit) we may use it to define a correspondence between points of $G(F)$ and points of $H(F)$. Indeed, ψ induces a bijective map ψ_{conj} from the conjugacy classes in $G(\overline{F})$ to those in $H(\overline{F})$ and because $\psi\sigma(\psi)^{-1}$ is inner, $\sigma \in \Gamma$, ψ_{conj} respects the action of Γ. Thus we have a bijection between the classes defined over F. We shall consider just those elements in G (or H) which are strongly regular in the sense that their centralizers are tori. The conjugacy class in $G(\overline{F})$ of a strongly regular element γ_G of $G(F)$ is defined over F. By a converse theorem of Steinberg for quasi-split groups, the image under ψ_{conj} of this class also contains F-rational elements. We say that any such element γ_H is *an image of* γ_G.

Recall that the stable conjugacy class of a strongly regular element in $G(F)$ (or $H(F)$) consists of the F-rational elements in its $\overline{F}$-conjugacy class. Thus the correspondence (γ_G, γ_H) provides an injective map of the set of stable conjugacy classes of strongly regular elements in $G(F)$ in the stable classes of such elements in $H(F)$ (this map is surjective only if G is quasi-split and thus F-isomorphic to H).

[1]Partially supported by NSF Grant 89-03313

Because the strongly regular elements in $G(F)$ are dense in $G(F)$ for the topology inherited from F, the correspondence just defined is sufficient to specify *transfer*. Recall that the stable orbital integral of $f \in C_c^\infty(G(F))$ (or $f \in \mathcal{C}(G(F))$) at strongly regular $\gamma \in G(F)$ is

$$\Phi^{st}(\gamma, f) = \sum_{\gamma'} \Phi(\gamma', f), \tag{1.1}$$

where the summation is over representatives γ' for the conjugacy classes in the stable conjugacy class of γ and $\Phi(\gamma', f)$ is the ordinary orbital integral

$$\Phi(\gamma', f) = \int_{G_{\gamma'}(F)\backslash G(F)} f(g^{-1}\gamma' g)\, d\bar{g}.$$

Here $G_{\gamma'} = \mathrm{Cent}(\gamma', G)$ and $d\bar{g}$ is the quotient of some fixed Haar measure on $G(F)$ by a Haar measure on the Cartan subgroup $G_{\gamma'}(F)$. If $\gamma' = g^{-1}\gamma g$, $g \in G(\overline{F})$, then $\mathrm{Int}\, g : G_{\gamma'} \to G_\gamma$ is defined over F: we require that the Haar measures on $G_\gamma(F)$ and $G_{\gamma'}(F)$ be related by transport under $\mathrm{Int}\, g$. The *transfer problem* for (G, H) is to show that for each $f^G \in C_c^\infty(G(F))$ there exists $f^H \in C_c^\infty(H(F))$ such that

$$\Phi^{st}(\gamma_H, f^H) = \begin{cases} \Phi^{st}(\gamma_G, f^G) & \text{if } \gamma_H \text{ is an image of } \gamma_G \\ 0 & \text{if } \gamma_H \text{ is not an image,} \end{cases} \tag{1.2}$$

for all strongly regular elements γ_H in $H(F)$.

Define $\Delta(\gamma_H, \gamma_G)$ for strongly regular $\gamma_H \in H(F)$ and $\gamma_G \in G(F)$ by

$$\Delta(\gamma_H, \gamma_G) = \begin{cases} 1 & \text{if } \gamma_H \text{ is an image of } \gamma_G \\ 0 & \text{otherwise.} \end{cases}$$

Then we may rewrite (1.2) as

$$\Phi^{st}(\gamma_H, f^H) = \sum_{\gamma_G} \Delta(\gamma_H, \gamma_G)\Phi(\gamma_G, f^G) \tag{1.3}$$

for all strongly regular $\gamma_H \in H(F)$. The summation is over representatives γ_G for the conjugacy classes of strongly regular elements in $G(F)$. If (1.1) or (1.2) is true we say that *f^G and f^H have Δ-matching orbital integrals.*

For F archimedean the transfer problem is solved, at least for the Schwartz functions [**S1**] or C_c^∞-functions bifinite under a maximal compact subgroup [**CD**]. For F nonarchimedean we consider first the problem of *local transfer at the identity*: given $f^G \in C_c^\infty(G(F))$ we are to find $f^H \in C_c^\infty(H(F))$ such that (1.2) holds for strongly regular γ_H near the identity in $H(F)$. This may be reformulated in terms of Shalika germs. The Shalika germ expansion yields

$$\Phi(\gamma, f) = \sum_{\mathcal{O}} \Gamma_{\mathcal{O}}(\gamma) a_{\mathcal{O}}(f)$$

for γ strongly regular near the identity, where the summation is over unipotent conjugacy classes $\mathcal{O}$ in $G(F)$, $\Gamma_{\mathcal{O}}$ is the Shalika germ for $\mathcal{O}$ and $a_{\mathcal{O}}(f)$ is the orbital integral of f along $\mathcal{O}$ (for some normalization of measure). We set

$$\Gamma_{\mathcal{O}}^{\mathrm{st}}(\gamma) = \sum_{\gamma'} \Gamma_{\mathcal{O}}(\gamma'),$$

where the summation is over representatives γ' for the conjugacy classes in the stable conjugacy class of γ; $\Gamma_{\mathcal{O}}^{\mathrm{st}}$ is the *stable germ* for $\mathcal{O}$. For transfer we will use instead the notation γ_G, $\mathcal{O}_G$, and so on. We write $\Gamma_{\mathcal{O}_G}^{(H)}$ for

$$\gamma_H \mapsto \begin{cases} \Gamma_{\mathcal{O}_G}^{\mathrm{st}}(\gamma_G) & \text{if } \gamma_H \text{ is an image of } \gamma_G \\ 0 & \text{if } \gamma_H \text{ is not an image.} \end{cases}$$

Then it follows readily that the pair (G, H) admits local transfer at the identity if and only if

(1.4) *for each unipotent conjugacy class $\mathcal{O}_G$ in $G(F)$, the transferred stable germ $\Gamma_{\mathcal{O}_G}^{(H)}$ is a linear combination of the stable germs $\Gamma_{\mathcal{O}_H}^{\mathrm{st}}$ for $H(F)$.*

For progress on (1.4) see the comments after (2.2).

We shall now assume local transfer at the identity for the centralizers of semisimple elements in $G(F)$ and deduce the full transfer on $G(F)$. The descent argument here is very simple.

First we formulate the assumption precisely. Call a semisimple element ϵ_H in $H(F)$ *an image* of the (semisimple) element ϵ_G in $G(F)$ if there exist a maximal torus T_G over F in G containing ϵ_G and an element x of $H(\overline{F})$ such that ${}_x\psi = \mathrm{Int}\, x \circ \psi : T_G \to H$ is defined over F and carries ϵ_G to ϵ_H. For strongly regular elements this coincides with the earlier notion of image. Applying a lemma of Kottwitz we may further choose x so that $H_{\epsilon_H} = \mathrm{Cent}(\epsilon_H, H)^\circ$ is quasi-split. Observe that ${}_x\psi : G_{\epsilon_G} \to H_{\epsilon_H}$ is an inner twist. It is not uniquely determined by ψ but its inner class is determined by the inner class of ψ, i.e., by

$\{\mathrm{Int}\, y \circ \psi : y \in H(\overline{F})\}$, and it is only the inner class that matters for the correspondence of F-rational strongly regular points.

Our assumption will be:

(1.5) *for each semisimple element ϵ_H of $H(F)$ the pair $(G_{\epsilon_G}, H_{\epsilon_H})$ admits local transfer at the identity.*

It is then immediate that:

(1.6) *we have local transfer for $(G_{\epsilon_G}, H_{\epsilon_H})$ at any central element of $H_{\epsilon_H}(F)$ and, in particular, at ϵ_H.*

Now we take $f^G \in C_c^\infty(G(F))$ and set

$$\Phi(\gamma_H, f^G) = \sum_{\gamma_G} \Delta(\gamma_H, \gamma_G)\Phi(\gamma_G, f^G).$$

We have to show that $\Phi(\ , f^G)$ is a stable orbital integral on $H(F)$. For this it is sufficient to show that it is *locally* a stable orbital integral on $H(F)$, i.e., for each semisimple $\epsilon_H \in H(F)$ there is $f_{\epsilon_H} \in C_c^\infty(H(F))$ such that

$$\Phi^{\mathrm{st}}(\gamma_H, f_{\epsilon_H}) = \Phi(\gamma_H, f^G)$$

for strongly regular γ_H near ϵ_H [**LS3**, Lemma 2.2.A].

The Harish-Chandra descent for orbital integrals says that near semisimple ϵ_G in $G(F)$ the stable orbital integral $\Phi(\gamma_H, f^G)$ is a sum of stable orbital integrals on the groups $G_{\epsilon'_G}(F)$, with ϵ'_G stably conjugate to ϵ_G. See [**LS3**, Section 1.5]. From (1.6) we conclude that near each semisimple ϵ_H in $H(F)$, $\Phi(\gamma_H, f^G)$ is a stable orbital integral on $H_{\epsilon_H}(F)$ and hence one on $H(F)$, and transfer for (G, H) follows.

§2 An outline of the general setting

An *endoscopic group* H for G is given as part of a *set of endoscopic data* for G (see [**LS1**]). This set also includes a group $\mathcal{H}$ which is "almost" the L-group ${}^LH$ of H and a suitable embedding of $\mathcal{H}$ in ${}^LG$. The group $\mathcal{H}$, rather than ${}^LH$, and the embedding arise naturally in two constructions of endoscopic data: the (T, κ)-construction associated with orbital integrals and the S_ϕ-construction associated with representations. In many cases, $\mathcal{H}$ is isomorphic to ${}^LH$ and transfer involves $H(F)$ itself. In general, however, we introduce a suitable central extension H_1 of H; in particular, $H_1(F) \to H(F)$ is surjective. Then for transfer we consider representations of $H_1(F)$ which act according to a character λ of $Z_1(F) = \ker(H_1(F) \to H(F))$ specified by our data. On the dual side, we consider the stable orbital integrals of those functions on $H_1(F)$ which transform under translation by $Z_1(F)$ according to the character λ^{-1}.

For the problems of local behavior (around a semisimple point) it makes no difference whether we work on $H_1(F)$ or on $H(F)$. Only in patching together the local results is passage to $H_1(F)$ necessary. This passage affects only a single term Δ_2 in the transfer factor which can be handled quite easily. See Section 4.4 of [**LS1**]. To simplify the exposition and save notation we will assume from now on that $\mathcal{H}$ is isomorphic to ${}^L H$ and so take $H_1 = H$.

We follow the steps in Section 1. First, there is a canonical map of semisimple conjugacy classes in $H(\overline{F})$ to such classes in $G(\overline{F})$ and this map respects Galois action (see [**LS1**]). The *strongly G-regular* classes of elements in $H(\overline{F})$ are those mapping to the strongly regular elements in $G(\overline{F})$. We have then a simple notion of *image* for stable conjugacy classes of strongly G-regular elements in $H(F)$ using the *inverse* of the canonical map on classes. *Transfer* is again specified by (1.3), i.e., by

$$\Phi^{st}(\gamma_H, f^H) = \sum \Delta(\gamma_H, \gamma_G)\Phi(\gamma_G, f^G),$$

now for all strongly G-regular elements γ_H of $H(F)$, and where now $\Delta(\gamma_H, \gamma_G)$ is the transfer factor of [**LS1**]. For *local transfer at the identity* we require (1.3) only for those γ_H near the identity and we may replace $\Delta(\gamma_H, \gamma_G)$ by a locally defined term $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$ which coincides with $\Delta(\gamma_H, \gamma_G)$ up to a constant.

To shorten the exposition we will assume F *nonarchimedean.* Suppose for now that G is *quasi-split.* There are various ways to describe $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$. See for example, [**H2**], [**LS2**], [**S2**]. We follow [**S2**]. Given $\gamma_H \in H(F)$ strongly G-regular and sufficiently close to the identity we shall fix an admisssible embedding of $T_H = \text{Cent}(\gamma_H, H)$ in G together with a set of a-data $\{a_\alpha\}$ for the image T_G of T_H. See [**LS1**, Sections (1.3), (2.2), (3.1)]. Let $\gamma_G \in T_G(F)$ be the image of γ_H under the embeding. As usual we parametrize the conjugacy classes in the stable conjugacy class of γ_G by $\mathcal{D}(T_G)$. Suppose $\gamma_G(\omega)$ is an element in the class attached to $\omega \in \mathcal{D}(T_G)$. Then

$$\Delta_{\text{loc}}(\gamma_H, \gamma_G(\omega)) = \kappa(\omega)\Delta_{\text{loc}}(\gamma_H, \gamma_G) \tag{2.1}$$

where κ is the character on $\mathcal{D}(T_G)$ determined by the endoscopic data underlying H [**LS2**]. Thus we have just to describe $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$. It is the product of

(i) a root of unity determined by the embedding and a-data (Δ_I in [**LS1**]),

(ii) the usual discriminant function (Δ_{IV} in [**LS1**])

and

(iii) a term indexed by orbits of the Galois group Γ in the set R_G of roots of T_G in G.

An orbit $\mathcal{O}$ makes a nontrivial contribution only if it is symmetric, i.e., $\mathcal{O} = -\mathcal{O}$, *and* it consists of roots outside H, i.e., roots not lying in the image of R_H under the map induced by our embedding of T_H in G. Then take $\alpha \in \mathcal{O}$. For γ_H sufficiently near 1 the term $\alpha(\gamma_G)^{1/2} - \alpha(\gamma_G)^{-1/2}$ is defined in the usual way. Moreover,

$$\frac{\alpha(\gamma_G)^{1/2} - \alpha(\gamma_G)^{-1/2}}{a_\alpha}$$

lies in the fixed field $F_{\pm\alpha} \subset \overline{F}$ of the stabilizer of $\pm\alpha$ in Γ. Let F_α be the fixed field of the stabilizer of α in Γ. Because $\mathcal{O}$ is symmetric, F_α is a quadratic extension of $F_{\pm\alpha}$. Let χ_α be the attached quadratic character of $F_{\pm\alpha}$. Then the contribution to $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$ from $\mathcal{O}$ is

$$\chi_\alpha\left(\frac{\alpha(\gamma_G)^{1/2} - \alpha(\gamma_G)^{-1/2}}{a_\alpha}\right).$$

For general G there is an additional contribution to $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$. First, strongly G-regular γ_H may be the image of no γ_G in $G(F)$. In that case,

$$\Delta_{\text{loc}}(\gamma_H, \gamma_G) = 0$$

for all strongly regular γ_G in $G(F)$. On the other hand, if γ_H is the image of some element γ_G then we may embed $T_G = \text{Cent}(\gamma_G, G)$ in G^*, the quasi-split inner form of G. Let γ_{G^*} be the image of γ_G under this embedding. Then γ_{G^*} is an image of γ_G in the sense of Section 1, and with H regarded as endoscopic for G^*, γ_H is an image of γ_{G^*}. The Local Hypothesis indicates how we are to write $\Delta_{\text{loc}}(\gamma_H, \gamma_G))$ in terms of $\Delta_{\text{loc}}(\gamma_H, \gamma_{G^*}))$. See [**LS1**], [**LS2**] for details.

To express local transfer at the identity in terms of Shalika germs, let $\mathcal{O}_G$ be a unipotent conjugacy class in $G(F)$ and define

$$\Gamma^{(H)}_{\mathcal{O}_G}(\gamma_H) = \sum_{\gamma_G} \Delta_{\text{loc}}(\gamma_H, \gamma_G)\Gamma_{\mathcal{O}_G}(\gamma_G)$$

with the usual notational conventions. Then local transfer at the identity amounts to showing:

(2.2) *for each unipotent conjugacy class* $\mathcal{O}_G$ *in* $G(F)$, *the transferred "*κ*-germ"* $\Gamma^{(H)}_{\mathcal{O}_G}$ *is a linear combination of the stable germs* $\Gamma^{\mathrm{st}}_{\mathcal{O}_H}$ *for* H.

Property (2.2) is known for $\mathcal{O}_G$ regular [**LS1**], $\mathcal{O}_G$ trivial (an easy consequence of well-known results) or, in many cases, $\mathcal{O}_G$ subregular [**H1**]. There are also complete results for specific groups, e.g., $SU(3)$ [**LS2**], [**H2**] and $GSp(4)$ [**H3**].

Transition to globally defined transfer factors, i.e., factors defined on all strongly G-regular elements, is quite subtle. First, we introduce a term (Δ_2 in [**LS1**]) which is a quasi-character on the Cartan subgroup $T_H(F)$ containing γ_H. This character is an analogue of the "ρ-shift" for real groups and, similarly, uses classification of the embeddings of the L-groups of Cartan subgroups in ${}^LG$. Data from this analysis also allows us to replace (iii) above with a globally defined term. Moreover, we do not use (2.1) and the Local Hypothesis separately but instead introduce a new unified term (Δ_1 in [**LS1**]). Now, however, only a *relative transfer factor* $\Delta(\gamma_H, \gamma_G; \overline{\gamma}_H, \overline{\gamma}_G)$ is well-defined (and canonical), for γ_H, $\overline{\gamma}_H$ any two strongly G-regular elements, with $\overline{\gamma}_H$ an image of $\overline{\gamma}_G$. But then we fix some such $\overline{\gamma}_H$ and $\overline{\gamma}_G$, specify $\Delta(\overline{\gamma}_H, \overline{\gamma}_G)$ arbitrarily and set

$$\Delta(\gamma_H, \gamma_G) = \Delta(\gamma_H, \gamma_G; \overline{\gamma}_H, \overline{\gamma}_G)\Delta(\overline{\gamma}_H, \overline{\gamma}_G).$$

See [**LS1**, Sections 3.7, 4.1]. This normalization fits well with adelic considerations [**LS1**, Section 6] and the problem of handling several inner forms of G simultaneously.

With $\Delta(\gamma_H, \gamma_G)$ defined we continue the program of Section 1. Let semisimple ϵ_H in $H(F)$ be an image [**LS1**] of, say, ϵ_G in $G(F)$. Again we may assume H_{ϵ_H} quasi-split. Then H_{ϵ_H} is endoscopic for G_{ϵ_G} [**LS3**, Section 1.4]. We assume the analogue of (1.5), i.e., local transfer at the identity for all pairs $(G_{\epsilon_G}, H_{\epsilon_H})$. To obtain local transfer at ϵ_H we need only observe the following property of transfer factors under translation by central elements. We shall state it for (G, H). The center Z_G of $G(F)$ is canonically embedded in the center of $H(F)$. If $z \in Z_G$ and γ_H is an image of γ_G then $z\gamma_H$ is an image of $z\gamma_G$. According to [**LS1**, Lemma 4.4.A] there is a character λ on Z_G such that

$$\Delta(z\gamma_H, z\gamma_G) = \lambda(z)\Delta(\gamma_H, \gamma_G)$$

for all z, γ_H. The proof of this requires a detailed analysis of the term Δ_2 in the transfer factor (see [**LS3**, Sections 3, 4]).

We now assume the analogue of (1.6), i.e., each pair $(G_{\epsilon_G}, H_{\epsilon_H})$ admits local transfer around ϵ_H. For $f^G \in C_c^\infty(G(F))$, consider the "normalized κ-orbital integral"

$$\Phi(\gamma_H, f^G) = \sum_{\gamma_G} \Delta(\gamma_H, \gamma_G)\Phi(\gamma_G, f^G)$$

for γ_H near ϵ_H. If ϵ_H is not an image then $\Phi(\gamma_H, f^G)$ vanishes near ϵ_H. Otherwise fix ϵ_G with ϵ_H as an image. Each orbital integral $\Phi(\gamma_G, f^G)$ is, by Harish-Chandra descent, locally an orbital integral on $G_{\epsilon'_G}(F)$ for some suitable ϵ'_G stably conjugate to ϵ_G. We need also descent for the transfer factor $\Delta(\gamma_H, \gamma_G)$. The main theorem of [**LS3**] is that

$$\Delta_G(\gamma_H, \gamma_G) = (\text{const})\Delta_{G_{\epsilon'_G}}(\gamma_H, \gamma_G)$$

for γ_H near ϵ_H, γ_G near ϵ'_G. (For F archimedean, this is true in the limit). The result is better stated in terms of relative transfer factors. Take $\overline{\gamma}_H$, $\overline{\gamma}_G$ near ϵ_H, ϵ'_G respectively, with $\overline{\gamma}_H$ an image of $\overline{\gamma}_G$, and set $\Theta = \Delta_G/\Delta_{G_{\epsilon'_G}}$. Then

$$\Theta(\gamma_H, \gamma_G; \overline{\gamma}_H, \overline{\gamma}_G) = 1. \tag{2.3}$$

The proof of (2.3) in [**LS3**] proceeds as follows. First we reduce easily to the case G quasi-split and $\epsilon'_G = \epsilon_G$ (see Section 3.1). Then we analyse in detail the cohomologically defined factors Θ_I, Θ_2 of Θ (Sections 3, 4). The only other factor in Θ possibly making a nontrivial contribution is Θ_{II} which is given by a simple explicit expression indexed by Galois orbits of roots. See (5.1); Θ_{II} is the right side of (5.1.1) and the final formulas for Θ_I, Θ_2 are also given. We find that only orbits of roots outside both G_{ϵ_G} and H may make a nontrivial contribution to Θ_I, Θ_2 or Θ_{II}. At the same time a long reduction argument allows us to assume all roots in G have the same length and that they take only the values ± 1 on ϵ_G while the coroots take only these values on the endoscopic datum s defining H (Section 5). Under such conditions we are able to compute the product of Θ_I and Θ_2 explicitly (see Theorem 6.3.C) and so compare orbit by orbit with Θ_{II}. We have, in particular, a product formula over all places which allows us to assume odd residual characteristic. The rest is a number theoretical computation (see Section 6.6).

We may now finish. The normalized κ-orbital integral $\Phi(\gamma_H, f)$ is a linear combination of such integrals for the groups $G_{\epsilon'_G}$ (see Sections 1.5, 1.7 of [**LS3**] for more precise information) and so we follow the argument for the stable case in Section 1. *Our conclusion is that what remains to prove transfer is the local problem (2.2) for Shalika germs.*

This very long analysis has further consequences. We mention two examples. From the study of the regular unipotent contribution we obtain in the p-adic case a formula for regular unipotent germs [**S2**] and in the real case a proof that the transfer factors defined here coincide with those introduced earlier in [**S1**]. From descent we verify some conjectures of Kottwitz [**K**] about extending transfer factors and the matching of orbital integrals from the "most regular" classes to all equisingular semisimple classes [**LS3**, Section 2].

REFERENCES

[**CD**] L. Clozel and P. Delorme, *Le théorème de Paley-Wiener invariant pour les groupes de Lie réductifs*, Inv. Math. **77** (1984), 427–453.

[**H1**] T. Hales, *The subregular germ of orbital integrals*, thesis, Princeton University.

[**H2**] *Orbital integrals on $U(3)$*, preprint.

[**H3**] *Shalika germs on $GSp(4)$*, preprint.

[**K**] R. Kottwitz, *Stable trace formula: elliptic singular terms*, Math. Ann. **275** (1986), 365–399.

[**KS**] R. Kottwitz and D. Shelstad, *Twisted endoscopy*, in preparation.

[**LS1**] R. Langlands and D. Shelstad, *On the definition of transfer factors*, Math. Ann. **278** (1987), 219–271.

[**LS2**] ________________, *Orbital integrals on forms of $SL(3)$, II*, Can. J. Math. **XLI** (1989), 480–507.

[**LS3**] ________________, *Descent for transfer factors*, to appear.

[**S1**] D. Shelstad, *L-indistinguishability for real groups*, Math. Ann. **259** (1982), 385–430.

[**S2**] *A formula for regular unipotent germs*, Astérisque **171–172** (1989), 275–277.