Forward premiums as unbiased predictors of future currency depreciation: a non-parametric analysis

YANGRU WU*

Department of Economics, West Virginia University, Morgantown, WV 26506-6025, USA

HUA ZHANG

Department of Finance, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

A large body of literature employing regression analysis has reported that the forward premium is not an unbiased predictor of future currency depreciation. Many studies argue that the forward market unbiasedness hypothesis may be falsely rejected due to biased parameter estimates. Possible sources of bias include: the existence of a time-varying risk premium, systematic forecast errors and measurement errors. This paper investigates whether the forward premium can predict the direction of change in the future spot exchange rate using a distribution-free, non-parametric approach. Our tests strongly reject the unbiasedness hypothesis and conclude that the forward premium contains either no information or the 'wrong' information about future currency depreciation. (JEL F31 G15). © 1997 Elsevier Science Ltd.

Over the past decade a considerable number of studies has been conducted on the efficiency of the forward foreign exchange market. Much of this research has focused on examining whether the forward exchange rate can predict the future spot exchange rate. It is widely believed that in an efficiently functioning market with risk neutral investors, speculation will drive excess profits down to zero and therefore the forward rate should be an optimal predictor of the future spot rate. Indeed, early studies typically run the ex post future spot exchange rate on the current forward rate and generally find that the slope coefficient is not significantly different from one. These results have been interpreted as evidence in support of forward market efficiency [see, for example Cornell (1977) and Frenkel (1981), among others].¹ These researchers employ ordinary least squares (OLS) techniques for estimation and hypothesis

*We would like to thank an anonymous referee and James R. Lothian (the editor) for helpful comments and suggestions. The usual disclaimer applies.
testing. In fact, it is now well documented that both spot and forward exchange rates follow unit-root processes and therefore classical inference on regression parameters is invalid.\textsuperscript{2}

Subsequent researchers transform variables by subtracting the current spot exchange rate from both sides of the level regression, and regress the realized change in the future spot exchange rate on the current forward premium. In sharp contrast with results obtained from the level regression, however, the estimated slope coefficient from regressing the future exchange rate depreciation on the forward premium is found to be reliably less than one and often not significantly different from negative one. This evidence is reported in Bilson (1981), Hansen and Hodrick (1983), Cumby and Obstfeld (1984) and Fama (1984), among others. It is also found to be quite robust across different currencies, data frequencies and sample periods by follow-up studies.\textsuperscript{3} One implication of the slope coefficient equal to minus one is that when the forward premium is 1% above its sample mean, the rate of appreciation of the foreign currency is 1% below its sample mean. This result is puzzling and has been viewed as an anomaly in international finance.

One natural interpretation of this empirical regularity, as proposed by Fama (1984) and further elaborated by Hodrick and Srivastava (1986), among others, is that the negative slope coefficient is due to the existence of a time varying risk premium. Fama decomposes the forward premium into the expected rate of depreciation and a risk premium and shows that under the assumption of rational expectations, the regression slope coefficient can be downward biased toward negative values if the risk premium is both negatively correlated with and more volatile than the expected currency depreciation.

A second explanation is provided by Froot and Frankel (1989) who demonstrate that the bias of the regression slope coefficient is caused by expectation errors rather than a risk premium.

Cornell (1989) offers a third interpretation by arguing that two types of measurement errors in the data can account for a substantial part of the bias in the slope coefficient. The first measurement error is that most studies simply use data on the bid or ask exchange rates or the average of them, while ignoring the fact that forward market participants bear transactions costs as reflected in the bid–ask spread. The second type of measurement error is that the forward and future spot exchange rates do not align perfectly. However, a recent study by Bekaert and Hodrick (1993) demonstrates that these two error-in-variables problems that Cornell raises are not particularly important. Bekaert and Hodrick also simulate the conditional variance of future currency depreciation using a GARCH-in-mean model and find that the omission of the conditional variance of future exchange rate depreciation as an explanatory variable in the regression does not contaminate the regression slope coefficient significantly.

From an econometric point of view, much of the discussion on the possible sources of rejection centers on a classical problem in a regression model, namely, the correlation between the disturbance term and the regressor. If the forward premium is sufficiently negatively correlated with the error term, the OLS estimate of the slope coefficient can be biased downward toward zero or
even negative values and therefore the null hypothesis can be rejected even if the forward premium might itself provide correct information about the direction of future spot exchange rate change. In this paper, we re-examine the unbiasedness hypothesis for six countries using a non-parametric approach. The novel feature of our tests is that they are distribution free and hence are more robust and possibly less sensitive to outliers. Our results strongly reject the null hypothesis for all US dollar exchange rates, thus reconfirming the regression test results reported in the literature. Tests with the cross exchange rates using the Deutsche mark and the Japanese yen as vehicle currencies, respectively, produce somewhat mixed results. Based on the asymptotic distribution, the forward premium appears to correctly predict the future currency depreciation for the mark/pound, mark/French franc and yen/pound pairs. However, when inference is drawn from the finite sample empirical distributions of the test statistics generated by bootstrapping, the unbiasedness hypothesis for all the cross exchange rates is violated as severely as for the US dollar exchange rates.

The remainder of the paper is organized as follows. The following section reviews the standard regression test for the unbiasedness hypothesis and motivates our alternative econometric techniques. Section II outlines the non-parametric testing procedures. The empirical results are reported in Section III. Section IV carries out some simple bootstrapping experiments to examine the small-sample behavior of the non-parametric test statistics. Some concluding remarks are contained in the final section.

I. The forward premium regression

The standard test of the forward market unbiasedness hypothesis is based on regressing the \textit{ex post} depreciation in future spot exchange rate on the current forward premium. That is:

\begin{equation}
    s_{t+k} - s_t = \alpha + \beta(f_{t+k} - s_t) + \epsilon_{t+k},
\end{equation}

where: $s_t$ is the home currency (say the US dollar) price of a unit foreign currency at time $t$ and $f_{t+k}$ is the home currency price at time $t$ of a unit foreign currency to be paid for and delivered at time $t+k$, all in natural logarithms. In (1), the forward premium is said to be an unbiased predictor of future currency depreciation if $\alpha = 0$ and $\beta = 1$. If the forward premium is orthogonal to the regression error, $\epsilon$, then the OLS estimate of the slope coefficient will be consistent and unbiased. If, in addition, $\epsilon$ is serially uncorrelated and homoscedastic, classical inference will also be valid. On the other hand, if $\epsilon$ does not satisfy this latter assumption, which is typically the case when the term to maturity of the forward contract, $k$, is longer than the sampling interval, appropriate techniques can be used to correct for the serial correlation and/or conditional heteroscedasticity, so that correct inference can still be obtained. Numerous studies have been carried out to estimate (1) and test for whether $\beta = 1$. The general finding is that the estimate of $\beta$ is reliably less than one and often significantly below zero.
While there is a consensus on the test results, there is less agreement on the possible sources of rejection. We review here two dominating interpretations that have been proposed in the literature.\(^5\)

In (1), the regression slope coefficient, \(\beta\), can be written as:

\[
\beta = \frac{\text{Cov}[(s_{t+k} - s_t), (f_{t,k} - s_t)]}{\text{Var}(f_{t,k} - s_t)}.
\]

Let \(\delta_{t+k} = s_{t+k} - E_t s_{t+k}\) be the expectation error and \(\rho_t = f_{t,k} - E_t s_{t+k}\) be the forward market risk premium.\(^6\) With these notations, the forward premium and the future spot rate change can be straightforwardly rewritten as follows:

\[
\begin{align*}
(3) & \quad f_{t,k} - s_t = \rho_t + E_t(s_{t+k} - s_t), \\
(4) & \quad s_{t+k} - s_t = (f_{t,k} - s_t) + \delta_{t+k} - \rho_t, \\
(5) & \quad s_{t+k} - s_t = E_t(s_{t+k} - s_t) + \delta_{t+k}.
\end{align*}
\]

Under rational expectations, \(\delta_{t+k}\) is orthogonal to any information available at time \(t\). With this assumption, Fama (1984) uses (3) and (5) to decompose the slope coefficient in (2) as follows:

\[
\beta = \frac{\text{Var}[E_t(s_{t+k} - s_t)] + \text{Cov}[\rho_t, E_t(s_{t+k} - s_t)]}{\text{Var}(f_{t,k} - s_t)}.
\]

It is clear from (6) that if the risk premium is sufficiently negatively correlated with the expected depreciation, then the slope coefficient, \(\beta\), can become negative.

On the other hand, Froot and Frankel (1989) do not assume rational expectations a priori. They employ (3) and (4) to decompose the slope coefficient as follows:

\[
\beta - 1 = \frac{\text{Cov}[\delta_{t+k}, (f_{t,k} - s_t)]}{\text{Var}(f_{t,k} - s_t)} - \frac{\text{Var}(\rho_t)}{\text{Var}(f_{t,k} - s_t)} + \frac{\text{Cov}[E_t(s_{t+k} - s_t), \rho_t]}{\text{Var}(f_{t,k} - s_t)}.
\]

The first term on the right hand side of (7) represents the bias caused by a systematic expectation error. If speculators do not have rational expectations, the expectation error, \(\delta\), will in general not be orthogonal to the current information set. In particular, if this term is correlated with the forward premium, a bias will arise which can make the slope coefficient to deviate from one. The second term on the right hand side of (7) displays the effect caused by the existence of a forward risk premium on the slope coefficient estimate. If such a risk premium is time-varying and correlated with the expected depreciation of the future spot exchange rate, the OLS estimate of the slope coefficient will be further contaminated. The regression test is in nature a test on a joint null hypothesis of rational expectations and risk neutrality. As neither the market expectation nor the risk premium is directly observable to the econometrician, if the null is rejected by the data, it is difficult to disentangle the main source of rejection without further information. Using survey data, Froot and Frankel measure the market expectation by the median survey responses and use this approximation to construct the expected depreciation of exchange
rates. They demonstrate that the bias can be primarily attributed to the systematic forecast error, which is negatively correlated with the forward premium, rather than to a forward market risk premium. However, as Ito (1990) argues, because market participants are highly heterogeneous with significant individual effects in their expectation formation, the usefulness of survey data may itself be in doubt.

II. Non-parametric tests

To avoid the possible econometric problem incurred in regression analysis, in this paper we abandon the regression framework and implement two alternative tests: Fisher's sign test and Wilcoxon's signed rank test. These are distribution free, robust tests for whether the forward premium has the correct prediction of the direction of change in the future exchange rate. This section briefly reviews the methodology.²

Our central interest is whether the forward premium has any predictive power at all. We use an example here to facilitate the exposition of the test procedures. Suppose that for the \( t \)-th observation in the data, one observes that \((f_{i,k} - s_i) > 0 \) \((< 0)\), and \((s_{i + k} - s_i) > 0 \) \((< 0)\), then the forward premium is said to correctly predict the direction of future exchange rate change. On the other hand, if the forward premium and the future spot rate change have opposite signs, the forward premium is said to wrongly predict the future currency depreciation. Our null hypothesis is that the forward premium contains no information about future spot rate change. If this hypothesis is true, then in a given sample the forward premium should correctly predict the future spot rate change for approximately 50% of the time, apart from sampling errors. On the other hand, the forward premium is said to contain the right (wrong) information if it predicts the future spot rate change for more (less) than 50% of the time. Therefore the issue involves testing whether this percentage is statistically significantly different from 50%. To this end, we define the following index function:

\[
I(t) = \begin{cases} 
1 & \text{if } (s_{i-k} - s_i) \text{ and } (f_{i,k} - s_i) \text{ have the same sign;} \\
0 & \text{if otherwise.}
\end{cases}
\]

Fisher's sign test is based on the following statistic:

\[
S = \sum_{t=1}^{T} I(t),
\]

where \( T \) is the total number of observations in sample. The exact distribution of the test statistic, \( S \), is a binomial distribution with parameters \( T \) and \( p \), where \( p \) is the percentage that the forward premium correctly predicts the future spot rate change. Under the null hypothesis that the forward premium has no predictive power, \( p = 0.5 \) When \( T \) goes to infinity, \( S \) converges to a normal distribution as follows:⁸
\[ Z_s = \frac{S - pT}{\sqrt{p(1-p)T}} = N(0,1), \text{ as } T \to \infty. \]

The sign test considers the difference in sign between the forward premium and the future currency depreciation, but not the magnitude of the difference. Intuitively, a larger forward premium (in absolute value) should have a higher predicting power for the direction of change in the future spot rate than a smaller forward premium and therefore should be more important. The signed rank test can incorporate the magnitude of the forward premium into calculation and it is this test that we will further apply to our data.

Let \( D_t \) be the absolute value of the forward premium:
\[ D_t = |f_{t,k} - s_t|. \]

Let \( R^+(D_t) \) denote the absolute rank of \( D_t \), the rank among \( D_1, D_2, ..., D_T \). Wilcoxon’s signed rank test is based on the following statistic:
\[ W = \sum_{t=1}^{T} I(t)R^+(D_t). \]

This test gives an observation with a larger forward premium (in absolute value) a higher weight than an observation with a smaller premium. The exact distribution of \( W \) under the null hypothesis cannot be expressed in closed form but finite-sample critical values are available in (Wilcoxon et al., 1970). Like the sign test statistic, \( W \) also has an asymptotic normal distribution:
\[ Z_w = \frac{W - \mu_w}{\sigma_w} \Rightarrow N(0,1), \text{ as } T \to \infty, \]
where
\[ \mu_w = E(W) = \frac{T(T+1)}{4} \quad \text{and} \quad \sigma_w = \text{Var}(W) = \left[ \frac{T(T+1)(2T+1)}{24} \right]^{1/2}. \]

In a sufficiently large sample, the standard normal distribution approximates the exact distribution reasonably accurately (see Randles and Wolfe, 1979). We shall draw inference based on the asymptotic distribution as well as the finite-sample empirical distributions generated by non-parametric bootstrapping.

The two tests described above complement each other. The sign test emphasizes the directions of forward premium and future spot rate change, while ignoring the magnitude of the change and is hence more robust to extreme values and measurement errors. On the other hand, the signed rank test takes into account the magnitude effects and is thus more powerful in detecting deviations from the null hypothesis.

**III. Empirical results**

We employ monthly observations on spot and 1-month forward US dollar prices of the British pound (BP), the Japanese yen (JY), the Deutsche mark (DM), the French franc (FF) and the Canadian dollar (C$). In addition to the
Forward premiums as unbiased predictors of future currency depreciation: Y Wu and H Zhang
dollar exchange rates, we also conduct the tests with cross exchange rates using
the Deutsche mark and the Japanese yen as vehicle currencies, respectively. Our sample starts in March 1973 when the Bretton Woods system broke down and ends in May 1993. All data are taken from the Harris Bank's *Weekly Review* and are drawn from those Fridays occurring nearest to the end of the calendar month. Cross exchange rates are constructed from the relevant dollar rates using the triangular arbitrage condition.

For ease of comparison, we reproduce the standard results by running regression (1) with our own data set and test for each of the following three hypotheses:

1. \( H_0: \beta = 1 \), against \( H_1: \beta < 1 \);
2. \( H_0: \beta = 0 \), against \( H_1: \beta < 0 \); and
3. \( H_0: \beta = -1 \), against \( H_1: \beta \neq -1 \).

To account for possible conditional heteroscedasticity in the error term, we construct Newey and West (1987) standard errors for the parameter estimates. We choose four lags, conforming to their \( T^{1/4} \) rule. Panel A of Table 1 reports the estimation and test results for the dollar exchange rates. It is observed that the slope coefficient estimates are uniformly negative, with the British pound the strongest. For all five currencies, the null hypothesis that \( \beta = 1 \) can be rejected at least at the 5% level in favor of the alternative hypothesis of \( \beta < 1 \) (using a one-sided test). For the British pound and the Canadian dollar, one can reject the hypothesis that \( \beta = 0 \) in favor of the alternative that \( \beta \) is negative. Finally, none of the currencies can reject the hypothesis that \( \beta = -1 \) at the 5% level. Only for the Japanese yen can the null be rejected at the 10% level. Our results are consistent with those reported in the literature and provide strong evidence against the unbiasedness hypothesis.

Panel B of Table 1 exhibits the estimation and test results for the Deutsche mark exchange rates. The evidence against the unbiasedness hypothesis is somewhat mixed here. It is interesting to note that the slope coefficient is estimated with the correct sign for the Japanese yen and the French franc. In particular, the hypothesis of \( \beta = 1 \) cannot be rejected at conventional significance levels for the French franc. Nevertheless, for the pound and the Canadian dollar, the evidence is as strong as for the dollar rates. Namely, one can reject the hypothesis of \( \beta = 1 \) at the 1% level for both rates, and cannot reject the hypothesis of \( \beta = -1 \) at conventional significance levels. As for the yen exchange rates, Panel C of Table 1 shows qualitatively the same results as for the dollar rates. All point estimates of the slope coefficients are negative, albeit their relatively small magnitudes.

Panel A of Table 2 presents results on the two non-parametric tests for the dollar exchange rates. We first apply the sign test to each of the five currencies. Columns 3 and 4 report Fisher's \( S \) and the standardized \( Z_s \) statistics, respectively. The \( Z_s \) statistics are all negative except for the Deutsche mark, implying that for those four currencies over the sample period more than 50% of the observations have the forward premium and future spot rate change carrying opposite signs. Under the null hypothesis that the forward premium contains no information, the \( Z_s \) statistic follows the standard normal distribution. Using
\begin{table}
\centering
\caption{Regression results}
\begin{tabular}{lcccc}
\toprule
Currency & $\hat{\alpha}$ & $\hat{\beta}$ & $t$-ratio & $t$-ratio \\
 & & & $H_0: \beta = 1$ & $H_0: \beta = 0$ \hline
 & & & $H_1: \beta < 1$ & $H_1: \beta < 0$ \hline
BP & $-0.007$ & $-1.716$ & $-3.623^{***}$ & $-2.289^{**}$ & $-0.955$
 & $(0.003)$ & $(0.750)$ & & & \\
JY & $0.004$ & $-0.143$ & $-2.258^{**}$ & $-0.283$ & $1.692^{*}$
 & $(0.002)$ & $(0.506)$ & & & \\
DM & $0.004$ & $-0.836$ & $-2.002^{**}$ & $-0.911$ & $0.179$
 & $(0.003)$ & $(0.917)$ & & & \\
FF & $-0.002$ & $-0.655$ & $-2.094^{**}$ & $-0.829$ & $0.436$
 & $(0.003)$ & $(0.790)$ & & & \\
C$\$ & $-0.003$ & $-1.363$ & $-5.161^{***}$ & $-2.977^{***}$ & $0.792$
 & $(0.001)$ & $(0.458)$ & & & \\
\midrule
B. Deutsche mark as the vehicle currency
BP & $-0.008$ & $-0.898$ & $-2.393^{***}$ & $-1.132$ & $0.129$
 & $(0.004)$ & $(0.793)$ & & & \\
JY & $0.002$ & $0.367$ & $-1.816^{**}$ & $1.053$ & $3.922^{***}$
 & $(0.002)$ & $(0.349)$ & & & \\
FF & $0.001$ & $0.935$ & $-0.194$ & $2.785$ & $5.765^{***}$
 & $(0.001)$ & $(0.336)$ & & & \\
C$\$ & $-0.008$ & $-1.319$ & $-2.380^{***}$ & $-1.354^{*}$ & $-0.327$
 & $(0.004)$ & $(0.974)$ & & & \\
\midrule
C. Japanese yen as the vehicle currency
BP & $-0.006$ & $-0.128$ & $-1.846^{**}$ & $-0.209$ & $1.428$
 & $(0.003)$ & $(0.611)$ & & & \\
FF & $-0.005$ & $-0.067$ & $-3.438^{***}$ & $-0.217$ & $3.005^{***}$
 & $(0.002)$ & $(0.310)$ & & & \\
C$\$ & $-0.005$ & $-0.020$ & $-1.819^{**}$ & $-0.037$ & $1.746^{*}$
 & $(0.003)$ & $(0.561)$ & & & \\
\bottomrule
\end{tabular}
\end{table}

Numbers in parentheses are Newey–West robust standard errors with four lags. $^{*}$, $^{**}$ and $^{***}$ denote significance at the 10%, 5% and 1% level, respectively.

A one-sided test, this null hypothesis can be rejected at the 1% level for the Japanese yen, at the 10% level for the French franc and at the 5% level for the Canadian dollar, in favor of the alternative hypothesis that the forward premium contains the wrong information in predicting the direction of future currency depreciation. For the British pound and the Deutsche mark the null cannot be rejected statistically in favor of either alternative hypothesis, thus
<table>
<thead>
<tr>
<th>Currency</th>
<th>Sample size $T$</th>
<th>Sign test</th>
<th>Signed rank test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$S$</td>
<td>$Z_s^a$</td>
</tr>
<tr>
<td>BP</td>
<td>242</td>
<td>118</td>
<td>-0.386</td>
</tr>
<tr>
<td>JY</td>
<td>242</td>
<td>100</td>
<td>-2.700***</td>
</tr>
<tr>
<td>DM</td>
<td>242</td>
<td>124</td>
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<tr>
<td>FF</td>
<td>242</td>
<td>111</td>
<td>-1.286*</td>
</tr>
<tr>
<td>CS</td>
<td>242</td>
<td>104</td>
<td>-2.186**</td>
</tr>
<tr>
<td>Pool</td>
<td>1210</td>
<td>557</td>
<td>-2.760***</td>
</tr>
</tbody>
</table>

A. US dollar as the vehicle currency

| BP | 242 | 135 | 1.800** | 15,754 | 0.965 |
| JY | 242 | 108 | -1.671** | 13,121 | -1.450* |
| FF | 242 | 132 | 1.414* | 16,937 | 2.051** |
| CS | 242 | 124 | 0.386 | 14,347 | -0.325 |
| Pool | 968 | 499 | 0.964 | |

B. Deutsche mark as the vehicle currency

| BP | 242 | 134 | 1.671** | 15,348 | 0.593 |
| FF | 242 | 125 | 0.514 | 15,423 | 0.662 |
| CS | 242 | 121 | 0.000 | 13,274 | -1.309* |
| Pool | 726 | 380 | 1.262 | |

C. Japanese yen as the vehicle currency

Both $Z_s$ and $Z_w$ follow $N(0,1)$ in large samples.

*, ** and *** denote significance at the 10%, 5% and 1% level, respectively.

concluding that the forward premiums contain no information for these two currencies.

The sign test does not require the observations to be identically distributed and can therefore be applied to the pooled sample of all five currencies in spite of their scale differences. This pooling greatly increases the sample size and hence improves the test power. Remarkably, the null hypothesis can be rejected at the 1% level in favor of the alternative that the forward premium has the wrong prediction for the pooled sample (see the last row of Panel A).

Next we take the magnitude effect into account and apply Wilcoxon’s signed rank test to each currency, using the absolute value of the forward premium to rank observations. The results are listed in Columns 5 and 6 of Panel A. All $Z_w$ statistics are negative and the null hypothesis can be rejected at the 1%, 10% and 5% levels for the Japanese yen, the French franc and the Canadian dollar, respectively, in favor of the alternative that the forward premium contains the wrong information. Notice that for the Deutsche mark, the sign
test statistic is positive but the signed rank statistic becomes negative. This implies that while the forward premium and future spot rate change have opposite signs for less than 50% of the sample, those observations with larger forward premiums carry higher weights.

It is interesting to compare these findings with the standard regression results of Panel A of Table 1. For the British pound, the regression test rejects the hypothesis of $\beta = 0$ at the 5% level in favor of the alternative that $\beta < 0$, while the non-parametric tests cannot reject the null that the forward premium contains no information. On the other hand, among the five currencies investigated, the non-parametric tests provide the strongest evidence against the null for the Japanese yen, but the regression test produces the weakest evidence for the yen. In particular, the hypothesis of $\beta = -1$ can be rejected at the 10% level by the regression test.

The non-parametric approach yields mixed results for the Deutsche mark exchange rates as can be seen from Panel B of Table 2. The sign test results show that, for the pound and the French franc, the forward premium has the correct prediction of the direction of change in future spot rate. The signed rank test reaches the same conclusion for the franc. On the other hand, both tests indicate that the forward premium predicts the currency depreciation in the opposite direction for the yen, and does not have predictive power for the Canadian dollar. When the four currencies are pooled, there does not exist significant evidence that the forward premium has predictive power.

Panel C of Table 2 presents the results for the three yen exchange rates. It is clear that the pound is the only case in which the forward premium has some predictive power for future spot rate change by the sign test. For the other two currencies, the forward premium either has no predictive power or predicts the future spot rate change in the wrong direction. The pooled test again shows no predictive power of the forward premium.

IV. Effects of small-sample bias

The non-parametric analysis of the last section is based on the asymptotic distribution of the test statistics. In this section, we examine how robust the results are when inference is based on the finite sample distributions. To this end, we conduct a simple bootstrapping experiment to estimate the exact distributions of the non-parametric test statistics. This simulation exercise is described as follows.

Our null hypothesis is that the forward premium does not have any predictive power at all for future spot rate change. Let

$$x_t = f_{t-k} - s_t, \quad y_t = s_{t+k} - s_t.$$ 

Under the null hypothesis, $s_t$ and $y_t$ are statistically independent. We assume that each one of them has a finite-order autoregressive representation:

$$\phi(L)x_t = \phi_0 + \xi_t,$$

$$\psi(L)y_t = \psi_0 + \zeta_t,$$
where $\phi_0$ and $\psi_0$ are constants; $\phi(L)$ and $\psi(L)$ are finite-order polynomials in the lag operator, $L$; and $\xi_i$ and $\zeta_i$ are mutually independent white noise processes.

Step 1: For each country pair, $\langle 14 \rangle$ and $\langle 15 \rangle$ are estimated with the data, where the autoregressive orders are chosen by the minimum Akaike information criterion (AIC). We obtain the parameter estimates as well as the fitted regression residuals, $\hat{\xi}_i$ and $\hat{\zeta}_i$. These are used as the data generating process.

Step 2: Randomly draw a sample of 242 error terms from the fitted residuals, $\hat{\xi}_i$, one observation at a time with replacement, and calculate 242 artificial observations of the forward premium, $x_i$, using the parameter estimates obtained in step 1, where initial values are set equal to those of the actual data. The artificial observations of the change of future spot exchange rate, $y_i$, are simulated in a similar way.

Step 3: Implement the sign test and signed rank test with the simulated observations of the forward premium and future spot rate change. Calculate the test statistics $Z_x$ and $Z_{\omega}$, using $\langle 10 \rangle$ and $\langle 13 \rangle$, respectively.

Step 4: Repeat steps 2 and 3 for 10,000 times to produce the empirical distributions for the test statistics under the null hypothesis.

We compute the $P$-value of each test statistic, which is defined as the percentage of the empirical distribution with values smaller than the test statistic calculated with the data. Notice that with this definition, a $P$-value smaller than 0.05 means that the null hypothesis that the forward premium has no predictive power can be rejected at the 5% level, in favor of the alternative hypothesis that the forward premium predicts the change in future spot rate in the wrong direction. On the other hand, a $P$-value greater than 0.95 indicates that the null hypothesis can be rejected at the 5% level, in favor of the alternative that the forward premium correctly predicts the direction of change in future spot rate. Table 3 presents the $P$-values of the test statistics based on the exact sample empirical distributions for each country pair. The $P$-values computed based on the large-sample distribution (standard normal) are also reproduced for comparison. We make several observations from Table 3.

First, from the results of Panel A, it is clear that for the dollar exchange rates, the finite-sample inference basically confirms the conclusions drawn from the asymptotic distribution, with the exception of the French franc where the null hypothesis can be rejected at the 10% level based on the standard normal distribution but cannot be rejected using the bootstrapping distributions. Both methods strongly suggest that the forward premium predicts the change of future spot rate in the wrong direction for the Japanese yen and the Canadian dollar, and does not have predictive power for the pound.

Second, from Panels B and C, it can be seen that based on the finite-sample inference, the forward premium has no predictive power for future spot rate change in general. Two exceptions are the DM/yen rate where the null hypothesis can be rejected at the 10% level by the sign test in favor of the alternative that the forward premium incorrectly predicts the future spot rate change, and the yen/Canadian dollar rate where the null hypothesis is rejected at the 10% level by the signed rank test.
<table>
<thead>
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<td>0.116</td>
</tr>
<tr>
<td>JY</td>
<td>-2.700</td>
<td>0.005</td>
</tr>
<tr>
<td>DM</td>
<td>0.386</td>
<td>0.684</td>
</tr>
<tr>
<td>FF</td>
<td>-1.286</td>
<td>0.188</td>
</tr>
<tr>
<td>CS</td>
<td>-2.186</td>
<td>0.004</td>
</tr>
</tbody>
</table>

A. US dollar as the vehicle currency

B. Deutsche mark as the vehicle currency

| BP      | 1.800    | 0.619           | 0.964        | 0.965  | 0.348           | 0.833 |
| JY      | -1.671   | 0.079           | 0.047        | -1.450 | 0.101           | 0.074 |
| FF      | 1.414    | 0.367           | 0.921        | 2.051  | 0.577           | 0.980 |
| CS      | 0.386    | 0.392           | 0.650        | -0.325 | 0.174           | 0.373 |

C. Japanese yen as the vehicle currency

| BP      | 1.671    | 0.588           | 0.953        | 0.593  | 0.230           | 0.723 |
| FF      | 0.514    | 0.367           | 0.696        | 0.662  | 0.411           | 0.746 |
| CS      | 0.000    | 0.460           | 0.500        | -1.309 | 0.087           | 0.095 |

\(Pval_{sv}\) — small-sample \(P\)-value, which is calculated based on the finite sample distribution generated by the bootstrapping method with 10,000 replications; \(Pval_{ws}\) — large-sample \(P\)-value, which is calculated under the assumption that \(Z_s\) and \(Z_w\) follow the standard normal distribution.

\(P\)-value is defined as the percentage of the distribution with values smaller than the test statistic calculated with the data. A \(P\)-value smaller than 0.05 means that the null hypothesis that the forward premium has no predictive power can be rejected at the 5% level, in favor of the alternative hypothesis that the forward premium predicts the change in future spot rate in the wrong direction. On the other hand, a \(P\)-value greater than 0.95 indicates that the null hypothesis can be rejected at the 5% level, in favor of the alternative that the forward premium correctly predicts the future currency depreciation.

Third, it is worth emphasizing that for the cross exchange rates there exist some discrepancies between the asymptotic and finite-sample inference. Specifically, the conclusions drawn from the limiting distribution that the forward premium correctly predicts the future spot rate change for the DM/pound, the DM/franc and the yen/pound pairs can no longer be obtained when inference is based on the exact sample bootstrapping distributions. Therefore, the unbiasedness hypothesis fails not only for the dollar exchange rates, but for cross exchange rates as well.
V. Conclusion

The hypothesis that the forward exchange rate premium is an unbiased predictor of future currency depreciation has been extensively tested by regression analysis in the literature. While there is no consensus on the possible sources of rejection of the null hypothesis, much of the discussion has focused on the regression problem caused by the correlation between the forward premium (the regressor) and the regression disturbance. This correlation makes the OLS estimate of the slope coefficient to be both downward-biased and inconsistent, thereby rejecting the null hypothesis.

This paper has adopted a distribution-free, non-parametric approach to re-examine the hypothesis. We find overwhelming evidence against the unbiasedness hypothesis for all five dollar exchange rates. While the tests provide limiting supporting evidence for the Deutsche mark and the Japanese yen exchange rates based on asymptotic inference, our bootstrapping experiment demonstrates that it can be primarily attributed to small-sample bias. Indeed, when inference is drawn from the exact sample empirical distributions of the test statistics, the unbiasedness hypothesis for the cross exchange rates is violated as severely as for the US dollar exchange rates. More notably, in many instances our tests are in favor of the alternative that the forward premium predicts the future spot rate changes in the opposite direction in a significant way. These results complement those of Bekaert and Hodrick (1993) based on regression analysis, and suggest that the forward premium contains either virtually no information or the wrong information in predicting future currency depreciation.

Notes

1. See (Hodrick, 1987) for a comprehensive survey of this literature.

2. The hypothesis that the slope coefficient is equal to one cannot be rejected when appropriate cointegrating vector estimators are employed. See, for example, Mark et al. (1993). However, because the regression is a cointegration regression, it only characterizes the long-run relationship between spot and forward exchange rates, but provides no information about the existence of a stationary risk premium.

3. Bekaert and Hodrick (1993) present evidence that for the Deutsche mark and the British pound, the negative slope coefficient is mainly produced by data since 1980. On the other hand, the slope coefficient is found to be positive and close to unity if one uses data from 1975 to 1980. They argue that there existed a structural break in January 1980. More recently, Wu and Zhang (1996) report results from switching regressions that the slope coefficient is negative during the periods when the forward premium is positive, but is positive and close to unity during those periods when the forward premium is negative.

4. Hodrick (1987, p. 28) has explained reasons for why the intercept term $\alpha$ can be allowed to be different from zero in an efficient forward exchange market. We shall therefore focus our discussion on the condition of $\beta = 1$.

5. Note that other attempts to understand the negative slope coefficient also include the Evans and Lewis (1992) ‘peso problem’ model, the McCallum (1994) policy reaction model, and the Mark et al. (1993) parametric structural time-series model.

6. We call $rp$, the risk premium following the convention in the literature, although it can be shown that the forward rate can deviate from the expected future spot rate under the assumption of risk neutrality. See, for example Domowitz and Hakkio (1985) and Kaminsky and Peruga (1990).
7. For a detailed description of these tests, see, e.g. Randles and Wolfe (1979).
8. Note that the null hypothesis that \( P = 0 \) or \( P = 1 \) cannot be tested because the distribution is degenerate. That is why we test the null hypothesis that the forward premium has no predictive power, instead of the usual unbiasedness hypothesis.
9. We have also applied the tests to the 3-month forward exchange rates and obtained similar results. Since those results are not of independent interest, they are not reported to save space.
10. We have also used other lags to compute the standard errors and find that the results are qualitatively the same.
11. The signed rank test cannot be applied to the pooled sample because it requires the observations to be drawn from an identical distribution (see Randles and Wolfe, 1979). This condition is not satisfied in the present case because of scale differences among currencies.
12. We have also computed the Schwarz information criterion (SBC). In most cases, the AIC and SBC choose the same lag lengths.
13. In drawing observations of the disturbance term, we choose to use the non-parametric bootstrapping, rather than the more standard parametric Monte-Carlo, because by using the former method we do not have to assume a distribution for the error term. A novel feature of the non-parametric tests introduced in this paper is that they do not rely on distributional assumptions.

References


