Endogenous growth and the welfare costs of inflation: a reconsideration

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Abstract

Many economists share the view that the welfare costs of moderate rates of money growth-cum-inflation are generally modest or small. This paper investigates the sensitivity of this result to alternative model specifications and behavioral assumptions. We first construct a monetary endogenous growth model with Romer's (1986) style of capital externality, and find that the real growth rate effect is substantially large, thus lending to much higher welfare costs than those in existing studies. To investigate the robustness of the result, we then conduct a series of experiments, by exploring the Romer model with an alternative specification of money demand and a class of other endogenous growth models. It is demonstrated, quite consistently, that our finding is confirmed in these models. © 1998 Elsevier Science B.V. All rights reserved

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1. Introduction

In the literature of money and growth there is a fairly widespread belief that the welfare costs of moderate rates of money growth (or equivalently inflation)
are generally modest or small. Using conventional welfare triangles analysis, Fischer (1981) obtains that the deadweight cost of an annual 10% money growth rate is about 0.3% of GNP, while Lucas (1981) offers a figure of 0.45%. In a general equilibrium real-business-cycle model, Cooley and Hansen (1989) report a cost of 0.387%, and they (1991) confirm basically the same finding in another model. Recently, Gomme (1993) extends the conventional analysis by incorporating endogenous growth, and finds, quite surprisingly, extremely small welfare costs, less than 0.03%, for a 10% money growth.

Academically, although this seems to be the dominant view, there nevertheless exist some different opinions and even criticism. İmrohoroğlu (1992), using a model in which optimizing households hold money to insure against unemployment, suggests that welfare triangles may underestimate the true costs of inflation by a factor of three or more. Howitt (1990) casts doubt on the ability of these relatively small estimates of welfare costs to account for the significance that people attach to inflation, who have shown a strong revealed preference for low rates of inflation, by their willingness to incur large costs to achieve a lower rate.

On the other hand, this belief of small welfare costs is at least inconsistent with the practices by many economic policy-makers in the world. For nearly two decades, indeed ever since the oil shocks of the 1970s, these policy-makers have been haunted by a fear of inflation. For the last decade or so, they have acted on this fear, tightening monetary policy, setting formal inflation targets, and taking various steps to squeeze the inflationary menace out of their economies. Consequently, inflation has been brought down to a low level by historical standards. For example, the average rate of inflation in the seven large industrial economies is now only 2.5%, close to its lowest rate for the past 30 years. Moreover, inflation has been consistently lower than expected in many countries, in particular in the United States, Britain and Japan. Despite the apparent success in these countries, inflation is still regarded as a main threat to their economies.

In this paper, we revisit the issue of the welfare costs of inflation in the context of endogenous growth. First, we develop an alternative endogenous growth model, in which perpetual growth arises from a positive capital externality as suggested by Romer (1986). Money enters the model through a cash-in-advance (CIA) constraint in consumption. As in Cooley and Hansen (1989), higher money growth-cum-inflation raises the opportunity cost of holding money and reduces the return to working. Through this channel money invokes not only level effect but also real growth rate effect. It is this latter effect, in contrast to the leisure effect in Gomme (1993), that significantly influences the welfare costs of inflation. As a result, it is found that the costs in our model are about 2–5% for a 10% money growth, which are substantially higher than 0.03% reported by Gomme.

Then, we conduct a series of experiments in an attempt to investigate the robustness of our finding. We first explore the monetary version of the Romer
model under an alternative specification of money demand. Specifically, money is introduced through a generalized CIA constraint or liquidity constraint, namely, all purchases of current consumption and a fraction of investment must be made using cash. We refer to this specification as CIA in everything. It is demonstrated that the costs of inflation are in the same order of magnitude as in the model with CIA in consumption.

We then go on to broaden our analysis to include a class of three other endogenous growth models. These models are the AK, two-sector, and Lucas models. For each model, those two specifications of money demand are applied, which we believe represent a reasonable cross section. We find, quite consistently, that our result of relatively large welfare costs for moderate rates of money growth is confirmed, albeit slightly different values of magnitude are produced across models. We conclude that Gomme's finding is probably due to the absence of external effects in the accumulation of human capital which are strongly advocated by Lucas (1988).

The remainder of the paper is organized as follows. Section 2 presents the economic environment and defines the perfect foresight competitive equilibrium. In Section 3, we conduct a balanced growth analysis. The model is then parameterized and calibrated, and its implied welfare results are given in Section 4. Section 5 considers the Romer model with CIA in everything, while Section 6 examines three other endogenous growth models. Finally, some concluding remarks are offered in Section 7.

2. Romer's model of endogenous growth and money demand

2.1. The environment

Consider an economy inhabited by a large number of infinitely lived homogenous individuals or households who have perfect foresight. Abstracting from the issue of population growth, the size of the population is constant and normalized to unity. A representative agent's instantaneous utility function in period $t$, $u_t$, depends on per capita consumption, $c_t$, and per capita leisure, $l_t$, or equivalently per capita work effort, $n_t$; i.e., $u_t = u(c_t, n_t)$. This function $u$ is assumed to be strictly concave and twice continuously differentiable, satisfying that $u_c > 0$, $u_n < 0$, $u_{cc} < 0$, $u_{nn} < 0$, $u_{nc} < 0$, and the Inada conditions. Two popular functional forms of $u$ will be utilized below.

In each period, the representative agent faces two constraints. The first is the budget constraint,

$$c_t + x_{kt} + \frac{m_t + i_t}{p_t} \leq w_t n_t + r_t k_t + \frac{m_t + \delta_t}{p_t},$$

(1)
where \(x_{kt}\) is investment purchase in physical capital, \(m_t\) is money balances at the beginning of period \(t\), \(p_t\) is the price level, \(w_t\) is the real wage rate, \(r_t\) is the rental price of capital during the period, and \(\tau_t\) is the size of transfers delivered by the government. The law of motion for physical capital is: \(k_{t+1} \leq (1 - \delta_k)k_t + x_{kt}\), where \(\delta_k\) is the depreciation rate on physical capital. The second constraint is a CIA constraint in consumption:

\[
c_t \leq \frac{m_t + \tau_t}{p_t}.
\]

On the production side, a representative firm has access to a common technology described by a function \(y_t = f(k_t, n_t, \bar{k}_t)\), where \(y_t\) and \(k_t\) represent output and capital per worker in period \(t\), respectively. The externality in capital accumulation, \(\bar{k}_t\), which equals \(k_t\) in equilibrium, is used to generate perpetual growth as in Romer (1986). More specifically, perpetual growth requires the function \(f\), which is also assumed to be well-behaved, to exhibit constant returns in \(k_t\) and \(\bar{k}_t\) taken together but diminishing returns in either input separately. As for the private inputs \(k_t\) and \(n_t\), \(f\) is postulated to be homogeneous of degree one when \(\bar{k}_t\) is held constant.

Profit maximization implies that factor prices are paid according to their marginal productivities:

\[
w_t = \frac{\partial f(k_t, n_t, \bar{k}_t)}{\partial n_t} \equiv f_n(k_t, n_t, \bar{k}_t),
\]

\[
r_t = \frac{\partial f(k_t, n_t, \bar{k}_t)}{\partial k_t} \equiv f_k(k_t, n_t, \bar{k}_t).
\]

Brief mention should be made that in the above setting, the government plays a fairly limited role. Its responsibilities are the provision of the money supply and then the redistribution of its seigniorage revenues in lump-sum terms to each individual. Hence, the government budget constraint is

\[
\tau_t = gm_t,
\]

where \(g\) is the exogenously given constant rate of money growth.

### 2.2. Perfect foresight competitive equilibrium

The representative agent seeks to maximize

\[
\sum_{t=0}^{\infty} \rho^t u(c_t, r_t),
\]

where \(\rho\) is her subjective rate of time preferences, subject to (1), (2), and the law of motion for physical capital, taking the initial capital stock, \(k_0\), and the path of
money supply as given. To maintain tractability and obtain analytical solutions, as a standard practice in the growth literature, we adopt the Cobb–Douglas form for the production function, \( f \): \(^1\)

\[ y_t = f(k_t, n_t, k_t) = Ak_t^\lambda n_t^{1-\lambda}k_t^{1-\lambda}, \tag{6} \]

where \( A \) is a positive scale parameter and \( \alpha \in (0, 1) \). It should be noted that increasing returns to scale arise in this model, and it is this potential factor that creates the wedge of the welfare implications between related studies and our model.

Let \( \theta_{1t}, \theta_{2t}, \) and \( \theta_{3t} \) be the Lagrange multipliers for constraints (1), (2) and the law of motion for physical capital, respectively, the dynamic optimum for this economy can be characterized, given the initial values of \( k_0 \) and \( m_0 \), by the following first-order conditions: \(^2\)

\[
\begin{align*}
\theta_{1t} &= \theta_{3t}, \\
- u_t(c_t, n_t) &= \theta_{1t} A(1 - \alpha)k_t^{\alpha}n_t^{-\alpha}k_t^{1-\alpha}, \\
\theta_{3t} &= \rho \left[ \theta_{1t+1} A\alpha k_t^{\alpha-1}n_t^{-\alpha}k_t^{1-\alpha} + \theta_{3t+1}(1 - \delta) \right], \\
\frac{\theta_{1t}}{p_t} &= \rho \frac{\theta_{1t-1} + \theta_{3t+1}}{p_{t+1}}, \\
\theta_{2t} \geq 0, \quad \frac{m_t + \tau_t}{p_t} - c_t \geq 0, \quad \theta_{2t} \left( \frac{m_t + \tau_t}{p_t} - c_t \right) = 0. \tag{7f}
\end{align*}
\]

along with the private budget constraint (1), and the transversality conditions:

\[ \lim_{T \to \infty} \rho^T \theta_{1T} m_T = \lim_{T \to \infty} \rho^T \theta_{2T} k_T = 0. \tag{7g} \]

Hence, a perfect foresight competitive equilibrium consists of the set of Eqs. (7), the private budget constraint (1), the government budget constraint (4), the equilibrium condition \( \hat{k}_t = k_t \), and the market-clearing conditions for the goods and money markets \(^3\)

\[
\begin{align*}
c_{t+1} &= Ak_t^\lambda n_t^{1-\lambda}k_t^{1-\lambda} + (1 - \delta)k_t, \tag{8a} \\
m_{t+1} &= m_t + \tau_t = (1 + g)m_t. \tag{8b}
\end{align*}
\]

\(^1\)The Cobb–Douglas function is a sufficient condition for the economy to follow what is known as a balanced growth path; see Chari et al. (1995).

\(^2\)A direct application of the Mangasarian sufficiency theorem yields that these necessary conditions are also sufficient.

\(^3\)As usual, the two market-clearing conditions are not independent. By Walras’ law, one can be derived from combining the private budget constraint (1) and the other.
To complete the model description, it remains to specify the utility function. In order to investigate the model's implied welfare costs of inflation, we, for the time being, adopt two popular functional forms of instantaneous utility, chiefly for comparative purposes. Moreover, the model is analyzed in a case where $\theta_{2t} > 0$ and the CIA constraint is binding. Indeed, it can be shown that the binding CIA case is the only case in equilibrium (see footnote 4 below).

3. Balanced growth analysis

In this section, we conduct a balanced growth analysis by focusing exclusively on the balanced growth equilibrium path, which is defined as the path along which extensive variables, $c_t$, $k_t$, $y_t$, $m_t$, $\theta_{1t}$, $\theta_{2t}$, and $\theta_{3t}$, grow at constant rates, while intensive variables such as leisure and work effort are constant. To accomplish this, we consider two specific utility functions: the first reflects indivisible labor, which has been used in a number of real business cycle models with money, see, e.g., Hansen (1985), Rogerson (1988) and Cooley and Hansen (1989) among others; and the second takes the usual form of constant elasticity of substitution (CES) between consumption and leisure.

3.1. The utility function reflecting indivisible labor

The instantaneous utility function is parameterized as

$$u(c_t, n_t) = \ln c_t - Bn_t,$$

(9)

where $B$ is a positive constant parameter. Notice that we adopt the logarithmic utility function not only because it is utilized in related studies, but also because it is the only formulation of preferences that is consistent with stationary labor supply in a growing economy when consumption and leisure/work effort are separable in the utility function (see Benhabib and Farmer, 1994).

Let $1 + \gamma = c_{t+1}/c_t > 0$ denote the constant (gross) growth rate of per capita consumption along the balanced growth path (BGP). Under the presumption of the binding CIA constraint, inflation is linked to money growth by the following relation:

$$(1 + \pi_t)(1 + \gamma) = 1 + g,$$

(10)

---

4Suppose the constraint is not binding, i.e., $\theta_{2t}$ is zero. Then, Eqs. (7a) and (7e) imply that $1 + \gamma = \rho/(1 + \pi)$, where $\pi$ is the inflation rate along the BGP; see the subsequent discussion. Using the money market equilibrium condition, we obtain $1 + g = \rho$, i.e., the money supply must grow at the negative rate of $(\rho - 1)$. Thus, if money grows at different rates (more likely to be positive), this leads to a contradiction. A similar argument can be made for the case of a CES utility function below.
where $1 + \pi_t \equiv p_{t+1}/p_t$ is the (gross) inflation rate. Evidently, (10) suggests that along the BGP, $\pi_t = \pi \forall t$. Next, it can be shown that along the BGP, consumption, capital stock, output, government transfers, and real money balances $(m/p)$ all grow at the same rate,

$$\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = \frac{(m/p)_{t+1}}{(m/p)_t} = 1 + \gamma.$$ (11)

Then, straightforward manipulation yields two equations with two unknowns, $(1 + \gamma)$ and $n$:

$$1 + \gamma = \rho[Az n^{1-x} + (1 - \delta_k)],$$ (12)

$$B(1 + g)[An^{1-x} + (1 - \delta_k) - (1 + \gamma)] = \rho(1 - z)An^{-x}.$$ (13)

Finally, to further reduce the dimensionality, we substitute (12) into (13) to produce

$$B(1 + g)[A(1 - \rho x) n^{1-x} + (1 - \rho)(1 - \delta_k)] = \rho(1 - z)An^{-x}.$$ (14)

Eq. (14) is a non-linear equation with one unknown, $n$, and its solution(s) can now be illustrated diagrammatically. Let $\Gamma(n)$ and $\Theta(n)$ be the expressions on the left- and right-hand sides of (14), respectively. It is easy to show that function $\Gamma$ is an upward-sloping concave curve, while function $\Theta$ is a downward-sloping convex curve, which are depicted in Fig. 1.\footnote{In Fig. 1, the intercept of the $\Gamma$ curve is given by $\Gamma_0 = B(1 + g)(1 - \rho)(1 - \delta_k) > 0$.} Obviously, the necessary and sufficient condition for the existence of a positive $n$, say, $\bar{n}$ requires that $B(1 + g)[A(1 - \rho x) + (1 - \rho)(1 - \delta_k)] > \rho(1 - z)A$, or the $\Gamma$ curve lies above the $\Theta$ curve when $n = 1$, which is assumed to be satisfied. Thus, one can easily establish a unique positive $n$ that solves (14).

Consequently, the balanced growth equations for other variables are given as follows. The consumption-capital ratio, $c/k$, and the investment-capital ratio, $x_k/k$, are

$$\frac{c}{k} = A\bar{n}^{1-x} + (1 - \delta_k) - (1 + \gamma),$$ (15a)

$$\frac{x_k}{k} = (1 + \gamma) - (1 - \delta_k).$$ (15b)

From the production function (6), the output-capital ratio is $y/k = A\bar{n}^{1-x}$. Finally, the common gross rate is determined by (12).\footnote{Technically speaking, a positive net rate of growth, $\gamma > 0$, is not warranted in (12). To ensure this, we further assume that $Azn^{1-x} + (1 - \delta_k) > 1/\rho$, requiring that $\bar{n}$ must be sufficiently high.}
Fig. 1. Determination of the work effort $n$ along the BGP: the case of the utility function reflecting indivisible labor.

To close this subsection, we consider the effects of an increase in the money growth rate, $\bar{g}$. It is conceivable that such a rise will shift the $\Gamma$ curve upward to the left in Fig. 1 without affecting the $\Theta$ curve, thus leading to unambiguously lower work effort $\bar{n}$. As a result, higher monetary growth depresses consumption, investment, output, and real money balances, which are all expressed as a percentage of capital; moreover, it also reduces the growth rate. In other words, money is found to be neither supernormal to real variables nor supernormal to the rate of economic growth. These results, which are generalizations of those in standard monetary models with zero or exogenous growth, accord reasonably well with intuition. An increase in monetary growth raises the opportunity cost of holding money, so people substitute capital away from money, leading to a lower marginal product of capital or a lower real rate of interest. Since the latter equals $(1 + \gamma)/\rho$, this results in a lower growth rate and a lower investment–capital ratio (from (15b)). A lower marginal productivity of capital also creates a lower derived demand for labor, and thus the work effort along the BGP declines, by factor complementarity $f_{\bar{w}} > 0$. Finally, as for the consumption–capital ratio, it is the direct result of a lower money holding because of the CIA constraint. It should be noted that exceedingly high money growth or hyperinflation could eventually push the economy into a path with a negative net rate of growth.

### 3.2. The CES utility function

The utility function takes the form

$$u_t(c_t, l_t) = \frac{\left(e^{\omega}l_t^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}, \quad \omega \in (0, 1), \quad \sigma \in (0, \infty),$$

(16)
where \( l_t = 1 - n_t \) is the amount of leisure that each individual takes. The (constant) intertemporal elasticity of substitution in consumption is then equal to \( 1/[1 - \omega(1 - \sigma)] \). Notice that when \( \sigma = 1 \) and \( n_t \) is 'sufficiently' small, (16) will be degenerated to the first utility function (9).

To aid the exposition, let \( d_1 \equiv -\omega(1 - \sigma) \), \( d_2 \equiv 1 + d_1 \), \( d_3 \equiv 1/d_2 \), and \( d_4 \equiv d_1d_3 \). It can be easily verified that the corresponding equations to (12) and (13) are

\[
(1 + \gamma)^{d_2} = \rho \left[ 4xn^{1-z} + (1 - \delta_k) \right],
\]

\[
(1 - \omega)(1 + g)(1 + \gamma)^{d_2}(1 - n)^{-\frac{1}{2}}[An^{-\gamma} + (1 - \delta_k) - (1 + \gamma)]
= \rho \omega(1 - \alpha)An^{-\gamma}. \tag{18}
\]

Finally, substituting \( (1 + \gamma) \) from (17) into (18), we have an analogue of (14):

\[
(1 - \omega)(1 + g)\rho^{d_2}(1 - n)^{-\frac{1}{2}}[An^{-\gamma} + (1 - \delta_k)]^{d_2}
\times \left[ [An^{-\gamma} - (1 - \delta_k)]^2 - \rho^{d_2}[4xn^{1-z} + (1 - \delta_k)]^{d_3} \right] = \rho \omega(1 - \alpha)An^{-\gamma}. \tag{19}
\]

Notably, the complication arises from the fact that \( d_3 \neq 1 \) (see the subsequent discussion) and the presence of the term \((1 - n)^{-\frac{1}{2}}\), where the latter implies that the marginal utility of leisure is no longer constant as in the first case. To solve (19), we have to specify the parameter values of \( \omega \) and \( \sigma \). Empirically, the intertemporal elasticity of substitution has been found to be far below unity (see, for example, Hall, 1988). Since the intertemporal elasticity of substitution is \( d_3 \) (= 1/[1 - \omega(1 - \sigma)]) and \( \omega > 0 \), this implies that \( \sigma > 1 \). Furthermore, Mehra and Prescott (1985) present a bulk of micro-evidence and argue that the coefficient of relative risk aversion lies between 1 and 2. It can be easily shown that this coefficient equals \((1 - \omega(1 - \sigma))\) ( = \( d_2 \)). Therefore, we have the parameter restrictions: \( d_1 \in (0, 1), d_2 \in (1, 2), d_3 \in (\frac{1}{2}, 1) \), and \( d_4 \in (0, 1) \).

Given the parameter restrictions, it is straightforward to show that there exists a unique and positive work effort \( n \) in (19). Similarly, along the BGP, the consumption–capital ratio, \( c/k \), the investment–capital ratio, \( x_k/k \), and the output–capital ratio, \( y/k \), can be obtained from (15a), (15b), and (6), respectively, whereas the gross rate of growth is given by (17). Moreover, it can be readily shown that the model under this case produces the same qualitative results as under the first case.

4. Model parameterization and welfare costs

4.1. Model parameterization

Whenever possible we choose parameter values based on growth observations of the US economy and the results of related studies. In order to make
comparisons with Cooley and Hansen (1989) and Gomme (1993) meaningful, we set the discount rate and the production parameters to the same values used in their studies. Specially, those values are \( \rho = 0.99, \alpha = 0.36, \) and \( \delta = 0.025, \) where the length of a period is taken as one-quarter. The balanced growth rate, \( \gamma, \) is set to 0.3542\%, the average quarterly growth rate of per-capita US GNP over the period 1954:1–1989:IV. The BGP money growth rate, \( g, \) is taken to be 0.014\%, close to the observed quarterly growth rate of per-capita US M1 over 1959:II–1989:IV.

The preference parameters are specified as follows. For the first utility function, we choose \( B = 2.86 \) as in Cooley and Hansen (1989). This value, in conjunction with others, implies that the scale parameter for production \( A = 0.2334 \) from combining (12) and (14). The BGP values of the endogenous variables obtained are \( c/k = 0.0789, \ x_k/k = 0.0285, \ n = 0.2975, \) and \( y/k = 0.1074. \) For the CES utility, we follow Gomme (1993) by setting \( \sigma = 3.1922 \) and \( \omega = 0.2281; \) these in turn produce \( A = 0.3168, \) along with the following BGP values of the endogenous variables: \( c/k = 0.0839, \ x_k/k = 0.0285, \ n = 0.1981, \) and \( y/k = 0.1124. \)

4.2. Welfare results

In this section, we derive a welfare cost measure and report the empirical results. To these ends, we rewrite the discounted sum of the utility at \( t = 0 \) as

\[
V(k_0, m_0; \lambda, 1 + g) = \sum_{t=0}^{\infty} \rho^t u(c_t + \lambda y_t, m_t),
\]

where \( \lambda \) is a lump-sum equivalent variation payment to the individual (expressed as a ratio to output) with the money growth rate \( (1 + g) \) for this regime. As in related studies, the welfare cost, \( \lambda, \) is defined in such a way that individuals would be as well off under this monetary regime as if there were no CIA constraint. The latter corresponds to the Pareto optimal situation. Let the superscript 'a' denote the respective value of that variable under an alternative monetary regime \( a. \) The welfare cost, \( \lambda, \) can then be obtained from the following equation:

\[
V(k_0, m_0; 0, 1 + g^a) = V(k_0, m_0; \lambda, 1 + g^a),
\]

where an asterisk, *, denotes the BGP value of a variable under the optimal allocation.\(^7\) When the utility function reflecting indivisible labor is used, (21) can

\(^7\)The optimal monetary growth rate, \( g^*, \) is one under which CIA is not binding. That is when \( \theta_z = 0 \) in (7f). It is straightforward to show that this optimal rule is determined by \( 1 + g^* = \rho \) with the utility function reflecting indivisible labor, and by \( 1 + g^* = \rho(1 + \gamma^*)^{\omega(1 - \sigma)} \) with the CES utility function.
be rewritten as (hereafter the initial \( k_0 \) is normalized to one)

\[
\frac{\ln(c/k)^* - Bn^*}{1 - \rho} + \frac{\rho \ln(1 + \gamma^*)}{(1 - \rho)^2} = \frac{\ln[(c/k)^* + \lambda(y/k)^*] - Bn^*}{1 - \rho} + \frac{\rho \ln(1 + \gamma^*)}{(1 - \rho)^2}.
\]

(22)

Similarly, when the CES utility is used, (21) is expressed as

\[
\left[\frac{(c/k)^*}{1 - \sigma}\right]^{\alpha(1 - \sigma)(1 - n^*/(1 - \sigma))} = \frac{1}{1 - \rho(1 + \gamma^*)^{\alpha(1 - \sigma)}}
\]

\[
= \left[\frac{(c/k)^* + \lambda(y/k)^*}{1 - \sigma}\right]^{\alpha(1 - \sigma)(1 - n^*/(1 - \sigma))} = \frac{1}{1 - \rho(1 + \gamma^*)^{\alpha(1 - \sigma)}}.
\]

(23)

Apparently, from either (22) or (23), it is observed that there exist three effects on the costs of inflation: consumption, leisure (or work effort), and the real growth rate. As discussed in Section 3, the consumption and the real growth rate effects move in the same direction, while the leisure effect moves in the opposite direction. Our empirical study below will assess each individual effect as well as the overall effect.

Table 1 presents the welfare costs as well as the BGP values of some key variables for the two model cases under five alternative money supply rules. The first rule corresponds to the optimal allocation, under which the economy experiences an annual money growth rate of \(-4\%\) for the utility function reflecting indivisible labor, and \(-5\%\) for the CES utility function. Two comments from Table 1 are in order. First and foremost, the most striking result is that the welfare costs of various money growth rates in either case are substantially higher than those obtained by Gomme (1993) in an alternative endogenous growth model; moreover, they are also considerably higher than those from similar models with zero or exogenous growth, such as in Cooley and Hansen (1989). Recall that the case with the utility reflecting indivisible labor is compatible with Cooley–Hansen’s, while the case with the CES utility is compatible with Gomme’s. From Table 1, a 10\% money growth results in welfare costs of 5.9763\% and 2.6525\%, respectively. By contrast, the cost is 0.3877\% in the Cooley–Hansen economy, while it is merely 0.03\% in the Gomme economy. These results are consistent across all monetary rules.

Our larger costs than the Cooley–Hansen’s are not surprising in that amongst other effects we identify an additional effect – the real growth rate effect. As for the difference between ours and those in Gomme, we also compute in Table 1 the contribution of each of the three factors that affect the costs of inflation in our model. By doing so, it is interesting to see the underlying factor that leads to the different results. Changing only leisure yields a benefit of 1.7426\% of income under a 10\% money growth regime, while changing consumption and the real growth rate generate costs of 1.3595\% and 3.0598\%,
Table 1
Welfare results of alternative annual money growth rates for the Romer model with CIA in consumption

<table>
<thead>
<tr>
<th>Annual money growth rate</th>
<th>Optimal</th>
<th>0%</th>
<th>10%</th>
<th>100%</th>
<th>300%</th>
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<tr>
<td>$1 + \gamma$</td>
<td></td>
<td>1.0</td>
<td>1.024</td>
<td>1.19</td>
<td>1.41</td>
</tr>
</tbody>
</table>

**Utility reflecting indivisible labor**

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<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>Output, $y/k$</td>
<td>0.1092</td>
<td>0.1085</td>
<td>0.1067</td>
<td>0.0909</td>
<td>0.0853</td>
</tr>
<tr>
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<td>0.0796</td>
<td>0.0784</td>
<td>0.0716</td>
<td>0.0646</td>
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<tr>
<td>Investment, $x_i/k$</td>
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<td>0.0289</td>
<td>0.0283</td>
<td>0.0245</td>
<td>0.0206</td>
</tr>
<tr>
<td>Hours worked, $n$</td>
<td>0.3054</td>
<td>0.3020</td>
<td>0.2944</td>
<td>0.2499</td>
<td>0.2073</td>
</tr>
<tr>
<td>Quarterly growth rate, $1 + \gamma$</td>
<td>1.0042</td>
<td>1.0039</td>
<td>1.0033</td>
<td>0.9995</td>
<td>0.9956</td>
</tr>
<tr>
<td>Annual inflation rate, $\pi$ (%)</td>
<td>5.5315</td>
<td>1.5503</td>
<td>8.5196</td>
<td>100.9399</td>
<td>302.2295</td>
</tr>
<tr>
<td>Welfare cost, $\hat{\lambda}$ (%)</td>
<td>1.7328</td>
<td>5.9763</td>
<td>38.4997</td>
<td>89.5697</td>
<td></td>
</tr>
<tr>
<td>Consumption effect (%)</td>
<td>-0.4475</td>
<td>1.4922</td>
<td>7.7524</td>
<td>14.1306</td>
<td></td>
</tr>
<tr>
<td>Leisure effect (%)</td>
<td>-0.6907</td>
<td>-2.2675</td>
<td>-10.7539</td>
<td>-17.9221</td>
<td></td>
</tr>
<tr>
<td>Real growth effect (%)</td>
<td>1.9826</td>
<td>6.8242</td>
<td>43.2035</td>
<td>97.5525</td>
<td></td>
</tr>
</tbody>
</table>

**CES utility**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output, $y/k$</td>
<td>0.1139</td>
<td>0.1132</td>
<td>0.1119</td>
<td>0.1036</td>
<td>0.0948</td>
</tr>
<tr>
<td>Consumption, $c/k$</td>
<td>0.0850</td>
<td>0.0845</td>
<td>0.0835</td>
<td>0.0771</td>
<td>0.0704</td>
</tr>
<tr>
<td>Investment, $x_i/k$</td>
<td>0.0289</td>
<td>0.0287</td>
<td>0.0284</td>
<td>0.0264</td>
<td>0.0243</td>
</tr>
<tr>
<td>Hours worked, $n$</td>
<td>0.2022</td>
<td>0.2003</td>
<td>0.1966</td>
<td>0.1743</td>
<td>0.1517</td>
</tr>
<tr>
<td>Quarterly growth rate, $1 + \gamma$</td>
<td>1.0039</td>
<td>1.0037</td>
<td>1.0034</td>
<td>1.0014</td>
<td>0.9993</td>
</tr>
<tr>
<td>Annual inflation rate, $\pi$ (%)</td>
<td>6.1500</td>
<td>1.4780</td>
<td>8.4639</td>
<td>99.3876</td>
<td>296.2887</td>
</tr>
<tr>
<td>Welfare cost, $\hat{\lambda}$ (%)</td>
<td>0.8740</td>
<td>2.6526</td>
<td>15.9747</td>
<td>36.3132</td>
<td></td>
</tr>
<tr>
<td>Consumption effect (%)</td>
<td>-0.4589</td>
<td>1.3595</td>
<td>6.9040</td>
<td>12.8184</td>
<td></td>
</tr>
<tr>
<td>Leisure effect (%)</td>
<td>-0.5942</td>
<td>-1.7426</td>
<td>-8.1797</td>
<td>-13.9996</td>
<td></td>
</tr>
<tr>
<td>Real growth effect (%)</td>
<td>1.0121</td>
<td>3.0598</td>
<td>17.7496</td>
<td>38.6382</td>
<td></td>
</tr>
</tbody>
</table>

respectively. This result of a much larger real growth rate effect is in contrast to Gomme’s larger leisure effect. It is therefore necessary to examine the robustness of our result in alternative models of endogenous growth and money demand, which are to be presented in the next two sections. Second, estimations of the welfare costs of inflation are slightly sensitive to utility functions used in actual experiments. We find that the costs are visibly higher in the utility function reflecting indivisible labor than those in the CES function. To understand this, naturally we need to examine the assumption of indivisible labor. Under such a hypothesis, individuals can work either some given positive number of hours or not at all, and they are not allowed to work an intermediate number of hours. In other words, all changes in total hours worked

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8. As in Gomme (1993), these costs and benefits do not sum to the total welfare cost of a 10% money growth, in that each change is considered in isolation by maintaining the other two variables at their BGP values under an optimal regime.
are due to changes in the number of workers, which, to a certain degree, is consistent with the stylized fact in the US economy. This is equivalent to an implicit introduction of unemployment costs of inflation into the model. As a result, Table 1 suggests that the relative magnitude of the leisure benefit has been trimmed under this specification, and therefore it is not surprising that higher welfare costs of inflation follow.9

In summary, our conclusion of significant welfare costs in Romer’s model of endogenous growth and money demand is robust not only to parameter perturbations, but also to two standard utility specifications. The impetus behind this finding stems from an important real growth rate effect. To further confirm our finding, we first investigate Romer’s model with an alternative type of money demand and then examine three other models of endogenous growth.

5. An alternative specification of money demand

In this section, we consider an alternative specification of money demand: a CIA specification in which all purchases must be made with currency, but in which cash has a differential productivity between consumption and investment purchases – CIA in everything.10

\[ c_t + \varepsilon x_t \leq \frac{m_t + r_t}{1 - \delta}, \]  

where \( 0 < \varepsilon < 1 \) is a constant. Under the assumption of CES utility, the balanced growth equations of the system are given by

\[ (1 + \gamma)^{1 - \sigma(1 - \sigma)} \left[ 1 - \varepsilon + \frac{\varepsilon(1 + g)}{\rho(1 + \gamma^{(1 - \sigma)})} \right] = \rho[n^{-1 - \omega} + (1 - \varepsilon)(1 - \delta)], \]  

\[ \frac{1 - \omega}{1 - \omega} \frac{c/k}{1 - n} = \frac{A(1 - \sigma)n^{-\sigma(1 + \gamma)^{(1 - \sigma)}}}{1 + g}, \]  

\[ 1 + \gamma = (1 - \delta) + x_k/k, \]  

\[ \frac{c/k}{1 + \gamma} + x_k/k = An^{1 - \omega}. \]

Given values of the model parameters and the policy variables, this system of four equations in four variables, \( c/k, x_k/k, 1 + \gamma, \) and \( n, \) can be solved to study the long-run effects of changes in monetary policy.

---

9In a recent effort to measure the costs of recession, Clark et al. (1994) introduce the risk of unemployment and present some evidence that the costs are much higher than those obtained using standard measures without unemployment risk.

Table 2
Welfare costs for the Romer model with CIA in everything
\((1 + g = 1.024, 10\%\ annual\ money\ growth\ rate)\)

<table>
<thead>
<tr>
<th></th>
<th>(\varepsilon = 0.2)</th>
<th>(\varepsilon = 0.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare cost, (\lambda) (%)</td>
<td>3.9011</td>
<td>5.1711</td>
</tr>
<tr>
<td>Consumption effect (%)</td>
<td>1.2774</td>
<td>1.1978</td>
</tr>
<tr>
<td>Leisure effect (%)</td>
<td>-1.8526</td>
<td>-1.9578</td>
</tr>
<tr>
<td>Real growth effect (%)</td>
<td>4.5145</td>
<td>5.9845</td>
</tr>
</tbody>
</table>

Inspection of Eqs. (25)–(28) reveals that the long-run reaction of the system to a change in monetary policy \((g)\) depends on the fraction of investment required to use cash, \(\varepsilon\). In what follows, we report results on two values of \(\varepsilon\): \(\varepsilon = 0.2\) and \(\varepsilon = 0.4\), where the former coincides with that used by Chari et al. (1995). The parameter values and the endogenous variables are: for \(\varepsilon = 0.2\), \(A = 0.3188\), \(c/k = 0.0845\), \(x_k/k = 0.0285\), \(n = 0.1978\), and \(y/k = 0.1130\); for \(\varepsilon = 0.4\), \(A = 0.3207\), \(c/k = 0.0850\), \(x_k/k = 0.0285\), \(n = 0.1975\), and \(y/k = 0.1136\). These values correspond reasonably well to their counterparts in the same model (Romer) with CIA in consumption.

We compute the welfare costs of a 10% monetary growth and the results are given in Table 2. The most notable feature is that the calculated costs are in the same order of magnitude as in the preceding model with CIA in consumption. For \(\varepsilon = 0.2\), the cost of a 10% monetary growth is 3.9011%, while for \(\varepsilon = 0.4\), it is 5.1711%. Moreover, these large costs are the result of a significant growth rate effect. These findings suggest that our conclusion of higher welfare costs in the Romer model is not sensitive to alternative specifications of money demand. However, a natural but more challenging question arises as to whether this conclusion also holds for other popular models of endogenous growth, to which the attention now turns.

6. Other models of endogenous growth and money demand

As in Chari et al. (1995),\(^{11}\) we report results for three other competing models of endogenous growth in the literature:

\(^{11}\)Throughout this section, we differ from Chari et al. in two main aspects. First, to make our results comparable with Cooley and Hansen (1989) and Gomme (1993), we abstract from the distinction between cash goods and credit goods. Second, from the period utility function (16), the importance of leisure relative to consumption is measured as \((1 - \omega)/\omega\), which is equal to 3.38 in our case (also in Gomme’s). By contrast, in Chari et al. (1995), this ratio is as high as 8.22. It is conceivable that these two features, in particular the latter, are important for the small inflation effects on growth in their model.
• A simple, one-sector model with a linear production function (AK);
• A generalization of the linear model that endogenizes the relative price of capital (two sector);
• A model with spillover effects in the accumulation of human capital (Lucas).

It should be mentioned that the last model introduces money into the Lucas (1988) model, which generalizes the model used by both Gomme (1993) and Chari et al. (1995) where the spillover effects in the accumulation of human capital are not modelled.\(^{12}\)

For the specifications of money demand, we retain the two considered earlier in this paper: CIA in consumption and CIA in everything. All together, these types produce six combinations of models. Rather than giving detailed derivations of each of the six models, we will describe the production functions in the three representative models and provide solutions only for the Lucas model with CIA in everything. Full details of the balanced growth equations for each model are available from the authors upon request.

For the two-sector model, the production function in the consumption sector is:\(^{13}\)
\[
c_t = Ak_{1t}^n h_t^{1-s},
\]  
(29)
and in the investment sector is
\[
x_{kt} = D(k_t - k_{1t}),
\]  
(30)
where \(k_1\) is the amount of capital in the production of the consumer goods and \(D\) is a constant.

For the Lucas model, the resource constraint in the manufacturing sector is
\[
c_t + x_{kt} = Ak_t^n (n_f t h_t)^{1-s} h_t^n,
\]  
(31)
and in the education sector is
\[
h_{t+1} = (1 - \delta_h)h_t + Dn_{ht}h_t,
\]  
(32)
where \(n_f\) is the effort devoted to production, \(n_{ht}\) is the effort devoted to human capital accumulation, \(h_t\) is the individual's human capital, \(\bar{h}_t\) is the average level of human capital, and \(\delta_h\) is the depreciation rate. The term \(\bar{h}_t^n\) captures the external effects of human capital.

Using the CES utility function of the form \(u(c, 1 - n_c - n_f) = [c^{\sigma}(1 - n_c - n_f)^{1-\sigma}]^{1-\sigma}(1 - \sigma)\), the balanced growth equations for the

\(^{12}\)Our investigation of the human capital model without the spillover effects largely reconfirms Gomme's results of very low welfare costs.

\(^{13}\)For the AK model, we follow Chari et al. (1995) by treating it as a special version of the Lucas model in which labor supply for production is inelastic.
Lucas model are

\[
(1 + \gamma_c)^{1-\sigma(1-\sigma)} \left[ 1 - \varepsilon + \frac{\varepsilon(1 + g)}{\rho (1 + \gamma_c)^{\sigma(1-\sigma)}} - \frac{\varepsilon(1 + g)(1 - \delta_k)}{1 + \gamma_c} \right].
\]

\[
= \rho \left[ \varepsilon A n_f^{1-\sigma} z^{x-1} + (1 - \varepsilon)(1 - \delta_k) \right],
\]

\[
(1 + \gamma_c)^{1-2(1-x+\eta)-\omega(1-\sigma)} = \rho \left[ D(n_f + n_h) + (1 - \delta_h) \right],
\]

\[
1 - \omega \frac{c/k}{\rho \omega} = \frac{A(1 - \lambda) n_f^{\sigma} z^{x-1} (1 + \gamma_c)^{\omega(1-\sigma)}}{1 + g},
\]

\[
1 + \gamma_c = (1 - \delta_k) + x_k/k,
\]

\[
1 + \gamma_h = (1 - \delta_h) + D n_h,
\]

\[
1 + \gamma_h = (1 + \gamma_c)^{1-2(1-x+\eta)}.
\]

\[
c/k + x_k/k = A n_f^{1-\sigma} z^{x-1},
\]

where \( z = k/h^{1-x+\eta(1-\sigma)/\sigma} \) and \( \gamma_c \) and \( \gamma_h \), respectively, are the net growth rates of consumption and human capital. Eqs. (33)–(39) define the system of seven equations in seven variables: \( c/k \), \( x_k/k \), \( z \), \( 1 + \gamma_c \), \( 1 + \gamma_h \), \( n_f \), and \( n_h \).

Our experiment proceeds as follows: for each of the six models, we compute solutions to the balanced growth equations, then use them as benchmarks to obtain solutions to the optimal allocation, and finally calculate the associated welfare costs when the annual monetary growth is 10%. The results of this experiment are contained in Table 3, in which the results for the Romer model are also included to facilitate comparisons. Since it can be shown that the growth rate is independent of the change in policy \( (g) \) in the two-sector model with CIA in consumption, the welfare cost is not computed. Alluded before, the AK model is treated as a special version of the Lucas model.

<table>
<thead>
<tr>
<th>Money demand specifications</th>
<th>CIA in consumption</th>
<th>CIA in everything</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth models</td>
<td>( \varepsilon = 0.2 )</td>
<td>( \varepsilon = 0.4 )</td>
</tr>
<tr>
<td>AK</td>
<td>2.0910</td>
<td>1.5129</td>
</tr>
<tr>
<td>Two-sector</td>
<td></td>
<td>0.8863</td>
</tr>
<tr>
<td>Lucas</td>
<td>1.7191</td>
<td>1.1135</td>
</tr>
<tr>
<td>Romer</td>
<td>2.6526*</td>
<td>3.9011</td>
</tr>
</tbody>
</table>

*When the utility function reflecting indivisible labor is used, this cost is 5.9763%.
The most obvious feature of our experiment lies in that higher welfare costs of inflation are generally found in other models of endogenous growth with alternative specifications of money demand. This is again in contrast to the small costs documented in Gomme (1993), in which only CIA in consumption is studied. To gain an intuitive insight into the difference, recall that in our monetary version of the Lucas model, the spillover effects in the accumulation of human capital are incorporated, while in the Gomme model such external effects are absent.\textsuperscript{14} On the one hand, it has been shown that in the latter with CIA in consumption, the relationship between inflation and growth is not monotone; see Jones and Manuelli (1990). Low costs are probably due to the three offsetting effects. On the other hand, it is well known that welfare costs can be large when there is a pre-existing distortion and inflation exacerbates that distortion. In our monetary version of the Lucas model (of course, this also applies to the Romer model), there is underinvestment and low growth, and inflation makes this worse.\textsuperscript{15}

It is also worth pointing out that the above interpretation helps explain why the costs in the Romer model are higher than any other models, since the adverse relationship between inflation and growth is monotone as shown in Section 3. Table 3 also suggests that the fraction $\delta$ is important in invoking the long-run effects of inflation. For some models, higher values of $\delta$ tend to induce higher costs, while for others the opposite is true.

7. Concluding remarks

This paper has reexamined the long-run effects of inflation in a framework with endogenous growth. Having used a variety of models with transactions demand for money, we find that the welfare costs of moderate inflation are large, ranging from half a percentage point to 5\% points for an annual 10\% monetary growth. This finding casts doubt on the negligible welfare costs suggested by the existing studies. As for the implications, it is supportive of a view that if monetary policy can affect the growth rate of the economy, its long-run welfare consequences will be large.

Since this study utilizes only a subset of available models of endogenous growth, we advise the readers not to draw a general conclusion from our finding which awaits for further corroboration. Future research also should incorporate some of the abstracted, but, important factors, such as transitional dynamic

\textsuperscript{14}There exists a substantial literature on the empirical plausibility of increasing returns in actual economies; see the references by Farmer and Guo (1994) for details.

\textsuperscript{15}We are indebted to an anonymous referee for suggesting this interpretation.
effects, the interaction with banking and financial sector and market imperfections.

References


