RATIONAL BUBBLES IN THE STOCK MARKET: ACCOUNTING FOR THE U.S. STOCK-PRICE VOLATILITY

YANGRU Wu*

Can rational stochastic asset bubbles help explain the excess volatility of stock prices? The bubble considered here is treated as an unobserved state vector in the state-space model and is easily estimated using the Kalman filter. I find that the bubble components estimated account for a substantial portion of U.S. stock prices, and the model does a credible job in fitting the data, especially during several bull and bear markets in this century. Much of the deviation of stock prices from the present-value model are captured by the bubble. (JEL G12, E44, C32)

I. INTRODUCTION

Many economists believe that stock prices are too volatile to be attributed to market fundamentals. In his seminal paper, Shiller [1981] reports that over the past century U.S. stock prices are five to thirteen times more volatile than can be justified by new information about future dividends. Research conducted by LeRoy and Porter [1981], independent of Shiller, also shows that the variability of actual stock price movements is too large to be explained by the present value of future earnings. While Shiller’s methodology was heavily criticized by subsequent researchers because of his assumption of stationarity in stock price and dividends and the small-sample bias of his estimators, his results on the excess volatility of stock prices have not been reversed.1

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Since the failure of the simple present-value model, economists have devoted a substantial amount of effort to seeking alternative model specifications to explain stock price volatility. One approach is to relax the assumption of a constant discount rate and allow investors to be risk averse, while maintaining the notion that stock prices are exclusively determined by the so-called market fundamentals. Variable discount rate specifications, however, provide only marginal support for the model in explaining stock price volatility. See, for example, Campbell and Shiller [1988a; 1988b] and West [1987; 1988].

Another approach that economists have taken to explain the data is to incorporate speculative bubbles into the model. Economists characterize rational asset bubbles as those generated by extraneous events or rumors and driven by self-fulfilling expectations. One implication of a rational bubble is that once a bubble is initiated, it will grow, in expectation, at a rate equal to the rate of discount and will eventually explode. The existence of speculative bubbles in financial markets has brought about heated debates and no consensus has been reached on whether bubbles are consistent with the rationality assumption on theoretical grounds.2 Empirically, partly due to the lack of power of testing

<table>
<thead>
<tr>
<th>ABBREVIATIONS</th>
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<tbody>
<tr>
<td>AIC: Akaike information criterion</td>
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<tr>
<td>CPI: Consumer Price Index</td>
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<tr>
<td>GMM: Generalized method of moments</td>
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<tr>
<td>MAE: Mean absolute error</td>
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<tr>
<td>RMSE: Root mean square error</td>
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<tr>
<td>S&amp;P: Standard and Poor’s 500 Index</td>
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procedures, general specification tests for stock market bubbles yield mixed results. For example, Rappoport and White [1993; 1994] and West [1987] reject the null hypothesis of no bubbles, while Dezhbakhsh and Demirguc-Kunt [1990] and Diba and Grossman [1988b] report the opposite results. Flood and Garber [1980], Hamilton and Whiteman [1985] and Hamilton [1986] criticize these bubble tests on the grounds that bubbles are observationally equivalent to regime changes in market fundamentals which are unobserved by the econometrician. Furthermore, Evans [1991] points out that tests for bubbles based on investigating the stationarity properties of stock prices and dividends are flawed. He demonstrates by Monte-Carlo simulations that an important class of rational bubbles cannot be detected by these tests even though the bubbles are explosive.

Froot and Obstfeld [1991] propose a particular type of bubble, called the intrinsic bubble. Unlike the bubbles traditionally defined, an intrinsic bubble depends exclusively on market fundamentals and not on extraneous events. They find significant evidence of such a bubble and demonstrate that incorporating an intrinsic bubble into the simple present-value model helps account for the long-run variability of the U.S. stock data. The intrinsic bubble specified by Froot and Obstfeld is unstable and implies an explosive price-dividend ratio. Furthermore, as their bubble is a deterministic function of dividends, once the bubble gets started, it will never burst as long as dividends remain positive. Froot and Obstfeld impose the free-disposal constraint on stock prices and allow only positive bubbles. Intuitively, however, it is difficult to imagine a situation in which market participants should always overvalue securities. In practice, tests for intrinsic bubbles are easily implemented only when dividends are assumed to follow a very simple process, for example, a geometric random walk. When a more general dynamic specification, such as an ARIMA\((p,1,q)\) process, is introduced for dividends, the test procedure for intrinsic bubbles becomes virtually intractable.

This paper specifies and estimates one type of rational bubble using the Kalman filtering technique. The bubble is stochastic and its parameterization is parsimonious and allows easy estimation. Although the bubble grows at the discount rate in expectation, as any rational bubble should, it can collapse and restart continuously. In particular, the bubble can oscillate from positive to negative. While in the real economy securities can be overvalued as investors are bullish, it is also likely that securities may be undervalued when market participants are more bearish. Therefore, while maintaining the free-disposal assumption, I do not impose, a priori, the non-negativity constraint for the bubble process in estimation and thereby allow for the possibility of a negative bubble. I assume that the log dividends follow a general ARIMA\((p,1,q)\) process. The model for stock prices with the bubble component, the dividend process and the bubble process are then expressed in the state-space form with the bubble being treated as an unobserved state vector. I estimate model parameters by the method of maximum likelihood and obtain optimal estimates of stochastic bubbles through the Kalman filter.

To summarize the results at the outset, I obtain a statistically significant estimate of the innovation variance for the bubble process. The estimated bubble components account for a substantial portion of the U.S. stock prices, especially during several major bull and bear markets in this century. In particular, significant estimates of positive bubbles appear during the 1960s bull market in which the bubble accounts for between 40% to 50% of the actual stock prices. Negative bubbles are found during the 1919–1921 bear market, in which the bubble explains between 20% to 30% of the decline in stock prices. To further demonstrate that this simple parametric specification of the rational bubble helps explain U.S. stock price volatility, two alternative models are also estimated, namely, the simple present-value model and Froot and Obstfeld's model with intrinsic bubbles. The goodness of fit of the two alternative models is compared with my model's. The root mean square error (RMSE) of the in-sample forecast

3. Ikeda and Shibata [1992] introduce a bubble that can depend on both time and market fundamentals. They demonstrate that their specification allows more dynamics for the bubble process. In particular, a bubble can be stochastically stable, saddle-point stable, or unstable.

4. Weil [1990] argues on theoretical grounds that it is possible for assets to be undervalued when the economy is in a bubbly equilibrium.
for the model presented here is 4% of the actual stock prices, while it is 22% with the intrinsic-bubble model and 40% with the simple present-value model. Using the in-sample forecast mean absolute error (MAE) as a metric, I find that the MAE of my model is 4%, while it is 18% under the intrinsic-bubble model and 33% under the simple present-value model. Overall, the rational stochastic bubble model does a credible job in characterizing U.S. stock market data.

The remainder of the paper is organized as follows. The model is specified and the estimation strategy is introduced in section II. Section III describes the data and conducts some preliminary diagnoses. Section IV presents the estimation results. Section V compares the performance of the current model with two alternative models. Some concluding remarks are provided in the final section.

II. MODEL SPECIFICATION AND ESTIMATION STRATEGY

Consider the standard linear rational expectations model of stock price determination,

\[(1) \quad [E_t(p_{t+1} + d_t) - p_t] / p_t = r\]

where:
- \(p_t\) = the real stock price at time \(t\);
- \(d_t\) = the real dividend paid at time \(t\);
- \(E_t\) = the mathematical expectation conditional on information available at time \(t\); and
- \(r\) = the required real rate of return, \(r > 0\).

Equation (1) is the arbitrage condition which states that the expected return from holding stocks should be equal to the required rate of return. It is straightforward to solve equation (1) and study the stock price dynamics with rational bubbles. However, one weakness of the specification in levels is that a negative bubble today implies that there is a positive probability that at some future time, the bubble will be large and sufficiently negative to make the stock price negative. As argued by Blanchard and Watson [1982] among others, if an asset can be disposed of at no cost, its price cannot be negative, and therefore negative bubbles can be ruled out, a priori, on theoretical grounds. To avoid the problem of obtaining negative theoretical prices, I express the present value model in terms of the natural logarithms of price and dividend. Following Campbell and Shiller [1988a], the log linear approximation of (1) can be written as follows:

\[(1') \quad q = \kappa + \psi E_t p_{t+1} + (1 - \psi) d_t - p_t\]

where:
- \(q\) = the required log gross return rate;
- \(\psi\) = the average ratio of the stock price to the sum of the stock price and the dividend, \(0 < \psi < 1\);
- \(\kappa = -\ln(\psi) - (1 - \psi) \ln(1 / \psi - 1)\);
- \(p_t = \ln(P_t)\); and
- \(d_t = \ln(D_t)\).

The unique forward-looking, no-bubble solution to (1'), denoted by \(p_t^f\), is given by

\[(2) \quad p_t^f = \lim_{i \to \infty} \left(\kappa - q\right) / \left(1 - \psi\right) + (1 - \psi) \sum_{i=0}^{\infty} \psi^i E_t (d_{t+i})\]

provided that the transversality condition is satisfied,

\[\lim_{i \to \infty} \psi^i E_t (p_{t+i}) = 0.\]

Equation (2) is the present value relation which states that the log stock price is equal to the present value of expected future log dividend streams. Notice that if the transversality condition is violated, then (2) is only a particular solution to (1'). The general solution to (1') takes the following form:

\[(3) \quad p_t = \left(\kappa - q\right) / \left(1 - \psi\right) + (1 - \psi) \sum_{i=0}^{\infty} \psi^i E_t (d_{t+i}) + b_t = p_t^f + b_t\]

where, \(\{b_t\}\) satisfies the following homogeneous difference equation:

\[(4) \quad E_t (b_{t+i}) = (1 / \psi)^i b_t \quad \text{for} \quad i = 1, 2, \ldots\]

5. I thank an anonymous referee for this suggestion.
In equation (3), the no-bubble solution \( p_t^* \) is exclusively determined by dividends and is often called the market-fundamental solution, while \( b_t \) can be driven by events extraneous to the market and is referred to as a rational speculative bubble. The existence of a bubble causes the actual stock price to deviate from its market-fundamental value.

Since the log dividends appear to be non-stationary, I specify the model in its difference form. Taking the first difference of (3) yields

\[
\Delta p_t = (1 - \psi) \sum_{i=0}^{\infty} \psi^i [E_t (d_{t+i})] - E_{t-1} (d_{t+e-1})] + \Delta b_t = \Delta p_t^* + \Delta p_r
\]

To obtain a parsimonious specification, I assume that the log dividends contain a unit root and can be approximated by an ARIMA(1,0) process as follows:

\[
\Delta d_t = \mu + \sum_{j=1}^{\delta} \varphi_j \Delta d_{t-j} + \delta_t
\]

where \( \delta_t \) is an i.i.d. error term and distributed \( N(0, \sigma^2_\delta) \).

The autoregressive order \( \delta \) in (6) is to be determined by the data. Equation (6) can be written in the companion form:

\[
(6') \quad Y_t = U + A Y_{t-1} + \nu_t
\]

where

\[
Y_t = (\Delta d_t, \Delta d_{t-1}, ..., \Delta d_{t-h+1})' \quad U = (\mu, 0, 0, ..., 0)'
\]

and \( \nu_t = (\nu_0, 0, 0, ..., 0)' \) are all \( h \)-vectors and

\[
\begin{bmatrix}
\varphi_1 & \varphi_2 & \varphi_3 & ... & \varphi_{h-1} & \varphi_h \\
1 & 0 & 0 & ... & 0 & 0 \\
0 & 1 & 0 & ... & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & ... & 1 & 0 \\
\end{bmatrix}
\]

is an \( h \times h \)-matrix.

Following Campbell and Shiller [1987], the solution to (5) can be obtained by using (6') as follows:

\[
(7) \quad \Delta p_t = \Delta d_t + M \Delta Y_t + \Delta b_t
\]

where:

\[
(8) \quad g = (1, 0, 0, ..., 0)
\]

is an \( h \)-row vector; and

\[
M = g A (I - A)^{-1} \left[ I - (1 - \psi) (I - \psi A)^{-1} \right]
\]

is an \( h \)-row vector and \( I \) is an \( h \times h \) identity matrix.

Assuming that the bubble process \( \{b_t\} \) is linear, equation (4) implies the following parametric form:

\[
(9) \quad b_t = (1/\psi) b_{t-1} + \eta_t,
\]

where the innovation \( \eta \) is assumed to be serially uncorrelated and have zero mean and finite variance \( \sigma^2_\eta \). It is also assumed that \( \eta \) is uncorrelated with the dividend innovation, \( \delta \), in equation (6).

A difficulty of estimating (7) arises because the bubble component is not observed. To facilitate estimation, I express the stock-price equation (7), the parametric bubble process (9) and the dividend process (6') in a state-space form. The bubble is treated as an unobserved state vector which can be estimated by the Kalman filtering technique. The Kalman filter is a recursive procedure for computing the optimal estimate of the bubble at each time period, based on the structural economic model and the observed data. The procedure is described in a technical appendix which is available upon request.

III. THE DATA

The data employed in this paper are those used in Cecchetti, Lam and Mark [1990] and are kindly provided by them. Real stock prices are the nominal Standard and Poor's (S&P) 500 index deflated by the Consumer Price Index (CPI). Real dividends are the nominal dividends for the S&P deflated by the CPI. All data are annual observations. The sample has 122 observations, covering from 1871 to 1992. Detailed data sources are described in the appendix to Cecchetti, Lam and Mark [1990].

The U.S. stock market experienced five long swings in this century. The first swing

6. The Kalman filtering technique has been widely used to estimate unobserved components of an economic model. For example, Burmeister and Wall [1982] use this technique to test for price-level bubbles of the German hyperinflation. Hamilton [1985] applies it to uncover unobserved market expectations of inflation. Wu [1995] employs this technique to study exchange-rate bubbles.
was during World War I. The S&P index lost roughly 40% in real terms between 1914 and 1918. The second long swing was the bull market between 1921 and early 1929, during which the stock index gained 290%. The market crashed in late 1929, causing the S&P index to drop 57% by 1933. The postwar boom began in the early 1950s and continued until the mid 1960s, pushing the market up by nearly 400%. And finally, the breakdown of the Bretton Woods system and the first oil-price shock initiated the 1970s bear market. The index lost 50% between 1973 and 1975.

The model is specified in its first-difference form in section II under the presumption that the log dividends are nonstationary and their first differences are stationary. To check whether this assumption is warranted, I apply both the augmented Dickey and Fuller [1979] test and the Phillips and Perron [1988] test to test for the existence of a unit root. The results are reported in Table I. As is seen, for the level data, the null hypothesis of a unit root is never rejected at conventional significance levels. This suggests that there exists at least one unit root in dividends. Proceeding to test for a second unit root, I difference the series and apply both test procedures to the transformed data. The test statistics reported in Table I all reject the null at the 1% level. Hence, there exists little evidence that would suggest a second unit root in dividends.

The stock-price equation depends crucially on the dynamics of the market forcing process. Section II assumes that the log dividends follow an ARIMA(1,1,0) process as specified in (6). To determine the autoregressive order $h$, I estimate the log dividend process by the maximum likelihood method for various choices of $h$, and compute both the Akaike information criterion (AIC) and the Schwartz information criterion for each $h$. The finding is that both criteria are at the minimum when $h = 2$. Thus in what follows, the model is estimated under the presumption that the log dividends follow an ARIMA(2,1,0) process.

IV. ESTIMATION RESULTS

The stock-price equation (7), the bubble process (9), and the dividend process (6') are jointly estimated by maximum likelihood through the Kalman filter. The model has six unknown parameters, $(\psi, \mu, \varphi_1, \varphi_2, \sigma_\eta, \sigma_\beta)$. Table II presents the point estimates as well as their standard errors. For the most part, the parameters are quite precisely estimated and are of plausible values. The parameters of the dividend process are close to those obtained from estimating the univariate dividend process (not reported). The standard deviation of the bubble innovation $\sigma_\eta$ significantly different from zero at the 1% level.

The ratios of the estimated bubbles to the actual stock prices (in percentage) are plotted in Figure 1, from which I make several observations. Firstly, it is evident that the stochastic bubbles fluctuate greatly and account for a substantial portion of the actual stock prices in the sample, especially during the major bull and bear markets in this century. Secondly, compared to their standard error estimates (not reported), the bubbles are found to be significantly positive at either the 1% or 5%
level in the 1960s when the stock market was extremely bullish. The results suggest that between 40% to 50% of the actual stock prices during this period might have been caused by speculative bubbles. Thirdly, it is interesting to note that while from a casual observation the 1920s were an extended bull market, the model produces negative estimates of bubbles for many of the years (except for 1926 and 1928), albeit these estimates are not statistically significant at conventional levels. Finally, although the model gives a negative estimate of the bubble for the 1987 stock market crash, the magnitude of the bubble is small and statistically insignificant.

V. COMPARISON TO OTHER MODELS

This section investigates the capability of the parametric specification of the bubble in explaining U.S. stock price volatility. Let $\hat{P}_t$ denote the fitted value of the log stock price at time $t$, using equation (3) and the parame-
TABLE III
Generalized Method of Moments Estimates of the Present Value Model and Dividend Process

<table>
<thead>
<tr>
<th>ψ</th>
<th>μ</th>
<th>φ₁</th>
<th>φ₂</th>
<th>Tmin(Jₚ)</th>
<th>p-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.950</td>
<td>0.008</td>
<td>0.171**</td>
<td>-0.110*</td>
<td>3.866</td>
<td>0.145</td>
</tr>
<tr>
<td>(0.572)</td>
<td>(0.012)</td>
<td>(0.050)</td>
<td>(0.047)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
a. Standard errors are in parentheses.
b. *Statistically significant at the 5% level.
**Statistically significant at the 1% level.
c. Tmin(Jₚ) is the sample size times the minimized criterion function and is distributed asymptotically as χ²(2).

Parameters and bubbles estimated in the previous section. The percentage in-sample prediction error is then approximated by (p̂ᵣ - pᵣ). To get a rough idea of how well the estimated model fits the data, I compute the in-sample root mean square error (RMSE) and mean absolute error (MAE) defined as follows:

\[
RMSE = \left( \frac{1}{T} \sum_{t=1}^{T} (\hat{p}_t - p_t) \right)^{1/2} \\
MAE = \left( \frac{1}{T} \sum_{t=1}^{T} |\hat{p}_t - p_t| \right).
\]

The performance of this model is compared to that of two alternative models. The first alternative is the simple present-value model, while the second is the present-value model with bubbles exclusively driven by dividends (intrinsic bubbles) as specified in Froot and Obstfeld [1991].

The simple present-value model to be estimated is equation (7) without the bubble component, together with the ARIMA(2,1,0) log dividend process (6'). The two equations are jointly estimated using Hansen’s [1982] generalized method of moments (GMM), allowing for fourth-order serial correlation and conditional heteroskedasticity in the Newey and West’s [1987] covariance matrix. Estimation results are presented in Table III.

For the intrinsic-bubble model, following Froot and Obstfeld, the dividend process is assumed to follow a geometric random walk with drift:

\[
\ln(Dₚ) = \alpha + \ln(D_{r-1}) + uₚ
\]

where \(uₚ \sim N(0, \sigma_u^2)\).

Using (12), the stock price equation (in levels) is obtained as follows:

\[
Pₚ = c₀Dₚ + c₁Dₚ^{1/λ}.
\]

The first term on the right-hand side of (13) is the market-fundamental value, while the second term is the intrinsic bubble which is exclusively a function of dividends. The parameter \(λ\) in (13) is the positive solution to the quadratic equation:

\[
(\sigma_u^2 / 2)λ^2 + \alpha λ - \rho = 0,
\]

where \(ρ\) is the instantaneous real rate of interest. The intrinsic-bubble model to be estimated is as follows:

\[
Pₚ / Dₚ = c₀ + c₁Dₚ^{1/λ} + vₚ,
\]

where \(vₚ\) is an error term.

Estimation of equation (12) with the data yields \(α = 0.012\) and \(σ_u = 0.1398\). The instantaneous interest rate \(ρ\) is estimated by the average return to the S&P 500 index, which is 8.06%. Using (14), I obtain \(λ = 2.323\). Equation (15) is estimated by least squares, where \(λ\) is fixed at 2.323, a priori. The standard errors of the parameter estimates are computed using Newey and West’s [1987] procedure which allows for fourth-order serial correlation and conditional heteroskedasticity. Estimation results are reported in Table IV. As in Froot and Obstfeld, a significant estimate of

7. See Froot and Obstfeld [1991] for the derivation of these results.
TABLE IV
Estimate of the Present Value Model with Intrinsic Bubble

<table>
<thead>
<tr>
<th>$c_0$</th>
<th>$c_1$</th>
<th>Adj $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.575**</td>
<td>808.600**</td>
<td>0.20</td>
</tr>
<tr>
<td>(1.395)</td>
<td>(311.108)</td>
<td></td>
</tr>
</tbody>
</table>

*Notes:*

a. The table reports least squares estimate of the following model:

$$P_t / D_t = c_0 + c_1 D_t \lambda + \nu_t$$

where the parameter $\lambda$ is fixed at 2.323. Numbers in parentheses report Newey-West standard errors allowing for fourth-order serial correlation and conditional heteroskedasticity.

b. Standard errors are in parentheses.

c. **Statistically significant at the 1% level.

FIGURE 2
Prediction Error of Stock Price: This Model

---

$c_1$, the parameter on dividends, is found, suggesting evidence of intrinsic bubbles.\(^8\)

Using the same methodology, I compute the in-sample prediction error for each period $t$, the RMSE, and the MAE for the simple present-value model and the intrinsic-bubble model, respectively.

8. While I use the same estimation strategy, my results are not quite the same as those of Froot and Obstfeld because their sample size is shorter and they use the Producer Price Index as the deflator.

Figures 2 through 4 plot the percentage prediction errors of the current model, the simple present-value model and the model with intrinsic bubbles, respectively. Several remarks can be made from these plots.

First, not surprisingly, the simple present-value model does a poor job in fitting the data, even though dividends are allowed to follow a more general ARIMA$(2,1,0)$ process rather than a simple random walk. The simple present value model does particularly poorly in
explaining the data in the 1870s and the 1960s periods during which its average prediction errors are approximately 50% and 60%, respectively. While the intrinsic bubbles seem to capture part of the movements of stock prices not explained by the present-value model, substantial variations are still left unexplained. In sharp contrast, much of the price movement unexplained by the present-value model can be reconciled with the rational bub-
TABLE V
Root Mean Square Error and Mean Absolute Error of Alternative Models

<table>
<thead>
<tr>
<th></th>
<th>Model in this Paper</th>
<th>Model with Intrinsic Bubbles</th>
<th>Simple Present Value Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE (%)</td>
<td>4.33</td>
<td>21.83</td>
<td>39.97</td>
</tr>
<tr>
<td>MAE (%)</td>
<td>3.66</td>
<td>18.09</td>
<td>32.69</td>
</tr>
</tbody>
</table>

Notes:
The table reports the in-sample root mean square error (RMSE) and mean absolute error (MAE), in percentage terms, for the three models. These are defined as follows:

\[
RMSE = \left( \frac{1}{T} \sum_{t=1}^{T} (\hat{p}_t - p_t)^2 \right)^{1/2}, \quad MAE = \frac{1}{T} \sum_{t=1}^{T} |\hat{p}_t - p_t|, 
\]

where \( p_t \) is the historical stock price at time \( t \) and \( \hat{p}_t \) is the fitted stock price at time \( t \) using an estimated model.

bles specified in this paper. This contrast can be seen from the much lower prediction errors produced by the current model when compared with the two alternatives throughout the sample.

Second, the current model does a credible job in characterizing the data during the 1960s bull market and the early 1970s sharp market decline. While the present-value model and intrinsic-bubble model, respectively, give average prediction errors of 66% and 24% for the boom from 1960 to 1966, the corresponding prediction error under the current model is only 4%. Following the break-down of the Bretton Woods system, the S&P index lost 50% in real value. For this sharp market decline, the present-value model’s average prediction error is 50% and the intrinsic-bubble model 21%, while this model again yields a mere 6%.

Finally, Table V reports the RMSE and MAE for the full sample for each of the three models. The RMSE of the in-sample forecast is 4% with the stochastic-bubble model presented in this paper, but it is 22% with the intrinsic-bubble model and 40% with the simple present-value model. The advantage of the stochastic-bubble model is also apparent if the MAE is used as a metric. Its MAE is 4%, as opposed to 18% with the intrinsic-bubble model and 33% with the present-value model.

VI. CONCLUSION
Economists have long been puzzled by the high volatility of historical stock prices as such volatility is difficult to explain in terms of market fundamentals. Speculative bubbles, or crowd psychology, have been viewed as possible sources that drive asset prices away from their fundamental values. There is no theoretic consensus as to whether bubbles can exist in a stochastic dynamic environment with rational economic agents. Empirically, general specification tests for bubbles yield mixed results, and even when the tests can reject the null hypothesis of no bubble, they still do not indicate the source of rejection.

I have proposed a specific type of rational bubble. The bubble specified is stochastic and thus able to burst and restart continuously. The specification is parsimonious and allows easy estimation. I found significant evidence of bubbles, which account for a substantial portion of U.S. stock prices. The model fits the data reasonably well, especially during several bull and bear markets in this century. Much of the deviation of U.S. stock prices from the simple present-value model can be explained by such rational stochastic bubbles if one is willing to accept the notion of bubbles.

REFERENCES


