Monopolistic competition, increasing returns to scale, and the welfare costs of inflation

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Abstract

This paper introduces monopolistic competition and increasing returns to scale into a monetary real business cycle (RBC) model to re-estimate the welfare costs of inflation. We first calibrate the model and show that it is capable of generating the observed aggregate fluctuations even when there are no shocks to the fundamentals. In particular, we demonstrate that this model matches the stylized U.S. business cycles facts as well as two more standard models. Then, we find that in this model the scale parameters and the intensity of competition significantly affects the welfare cost of inflation. Specifically, the cost is considerably higher than that in a standard RBC model with competitive markets and constant returns. Moreover, the stronger the increasing returns and the less intense the competition, the higher the welfare costs. These results are

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### 1. Introduction

The cash-in-advance economies of Lucas (1980,1984) and Lucas and Stokey (1983,1987), under the maintained hypothesis of competitive markets and constant returns to scale, have been widely used in monetary theory. One empirical application concerns the assessment of the welfare costs of inflation, and there exists a substantial literature on it. Within such a framework, estimates of the welfare costs of moderate inflation are generally modest or small. Cooley and Hansen (1989) report a cost of 0.387% of GNP for a 10% annual money growth rate. Later, Cooley and Hansen (1991) confirmed basically the same finding in another model with cash and costless credit and the presence of labor and capital taxation. Such a magnitude is also consistent with other estimates. Using welfare triangles, Fischer (1981) obtains a cost of 0.3%, while Lucas (1981) offers a figure of 0.45%.

Several subsequent researchers have attempted to re-address this issue by considering alternative specifications in the cash-in-advance economy. Unfortunately, they generate either incompatible or conflicting results. Among them, two papers, which take different routes, have reported quite different results. The first paper, by Gomme (1993), extends the Lucas-Stokey (1983) cash-in-advance economy to include endogenous growth. With an endogenous labor-leisure trade-off, Gomme obtains, quite surprisingly, extremely small welfare costs for various monetary growth rates. For example, a 10% money growth rate results in a welfare cost of less than 0.03% of income.\(^1\) The second paper, by Gillman (1993), modifies the Lucas-Stokey economy by specifying an exchange function through which the consumer decides whether to use cash or costly credit to purchase a good. It is shown that the consumer faces higher welfare costs than 0.4% in standard cash-in-advance economies. The magnitude is as high as 2.19% for a 10% monetary growth rate in this costly credit economy.\(^2\)

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\(^1\) Gomme argues that this result is largely due to the endogenous labor-leisure choice. It should be noted, however, that Gomme studies the welfare costs of inflation in a monetary version of the Lucas (1988) model with endogenous growth but omits the spillover effects in the accumulation of human capital. In a recent paper, Wu and Zhang (1998) reexamine this issue by considering such external effects and find that the costs are substantially higher than those documented by Gomme.

\(^2\) Den Haan (1990), in a model requiring time for cash exchange as well as for credit-type exchange, reports a comparatively high cost of 4.68% for the economy going from 0% to a 5% monetary growth.
All these studies, however, assume perfect competition, constant returns to scale and price taking behavior by the producers and consumers. This paper attempts to study the welfare cost issue in a framework with two new elements – monopolistic competition and increasing returns to scale.\(^3\) In such a framework, a technical issue needs to be addressed at first. It is known that the underlying economy in a standard monetary RBC model can be characterized by a saddle-path equilibrium. Under the alternative hypothesis of monopolistic competition and increasing returns to scale, this property can only be maintained if increasing returns are mild, such as in a real model by Hornstein (1993).\(^4\) However, when increasing returns are sufficiently strong, the above departure from the standard paradigm may alter the stability of the steady state and lead to potentially more radical implications. More specifically, such a model, as in Benhabib and Farmer (1994), displays an indeterminate steady state (i.e., a sink), and one may exploit this indeterminacy to generate a model of aggregate fluctuations that is solely driven by self-fulfilling beliefs, also known as sunspots or animal spirits in the literature. Farmer and Guo (1994) show that when the Benhabib and Farmer (1994) model is calibrated to the post-War U.S. data, not only is it a theoretical curiosity for sunspots to drive business cycles, but it occurs well within the range of parameter values that are empirically realistic and plausible. In other words, it is possible for purely extrinsic uncertainty to induce the magnitude of fluctuations that we observe.

In our model, there are a continuum of monopolistically competitive intermediate goods producers who face increasing returns technologies. As in recent studies of this type, increasing returns are allowed to vary to a particular stage where indeterminacy may occur. In this case, even if there is no fundamental uncertainty whatsoever, economic aggregates may still undergo irregular fluctuations as a direct result of the self-fulfilling beliefs of rational forward looking individuals. We show that this economy captures reasonably well the stylized facts of the U.S. business cycles as the standard monetary RBC model and a model with mild increasing returns and a saddle point.

\(^3\)Our use of noncompetitive markets and increasing returns economies is motivated by some of the recent empirical studies. For example, Hall (1986, 1988, 1990) suggests that increasing returns and monopoly power may better describe the real-world economies, which can potentially account for a number of macro issues, such as the observed correlation of the Solow residuals with some measures of aggregate demand shifts. Moreover, empirical results suggesting the presence of monopoly markups or externalities are also obtained in the literature, for instance, Domowitz et al. (1988), Baxter and King (1991), and Caballero and Lyons (1992). Indeed, there are a large number of business cycle models that include an imperfectly competitive element; see the citations by Farmer and Guo (1994) for details.

\(^4\)In Hornstein’s model, increasing returns arise from the introduction of an ‘overhead cost’ component, rather than the usual assumption that the rate of return in production sums to more than one.
We then derive a closed-form welfare cost function, and find that both the scale parameters and the intensity of competition significantly affect the welfare cost function. One of the main results of the current paper is that the welfare cost in this model with monopolistic competition and increasing returns is considerably higher than that in the Cooley–Hansen (1989) model with competitive markets and constant returns. In terms of magnitude, a 10% money growth leads to a welfare cost of 3.4% in our model as opposed to 0.4% in the Cooley–Hansen model. Moreover, the stronger the increasing returns, the higher the welfare cost; and the less the intensity of competition among firms, the higher the welfare cost. These findings also demonstrate that the quantitative effects of anticipated inflation may have been substantially underestimated if the underlying economy exhibits characteristics of non-standard features.

To investigate the robustness of our results, we go on to examine them with respect to a number of model specifications. First, we consider an alteration in the preferences of the representative agent to capture the more realistic feature of a varying marginal utility of leisure. Second, we expand our form of money demand to also include firms’ investment in the cash-in-advance constraint. Finally, we modify our model by using a shopping-time transactions technology of money demand. It is then found that our conclusion of higher welfare costs in the paradigm of monopolistic competition and increasing returns is confirmed under each variant, thereby suggesting that our results are quite robust and general.

The remainder of the paper is organized as follows. Section 2 presents the economic environment and defines the stationary equilibrium. In Section 3, we study the dynamics and expectational shocks. The model is then parameterized and calibrated, and is subject to a number of diagnoses in Section 4. Section 5 reports the welfare costs of inflation for our basic model specification and compares them with those implied by two more standard models. The welfare costs in models with intermediate parameter values are also studied. Section 6 examines three alternative specifications. Finally, some concluding remarks are offered in Section 7.

2. A basic model with monopolistic competition and strong increasing returns

2.1. The economic environment

The model economy is populated by a large number of individuals, each with the same preference ordering. Utility is derived from consumption, \( c_t \), and leisure, \( \ell_t \), or equivalently work effort, \( h_t \). (Variables are expressed in per capita terms.) The utility function is time separable and has a constant rate of intertemporal preference, \( \rho \)

\[
E_0 \sum_{t=0}^{\infty} \rho^t U(c_t, h_t), \quad \rho \in (0,1),
\]  

(1)
where $E$ is the mathematical expectation operator. This function $U$ is assumed to be strictly concave and twice continuously differentiable, satisfying $U_r > 0, U_h < 0, U_{cc} < 0, U_{ch} \leq 0, U_{hh} < 0$, and the Inada conditions.

The representative agent enters period $t$ with physical capital, $k_t$, and nominal cash balances, $m_t$. As is standard practice in the literature, we assume that in the beginning of the period, the state-of-the-world is revealed; namely, the current-period productivity shock, $Z_t$, and gross per-capita growth rate of money, $g_t$, are revealed. The government’s only role involves controlling the money supply process: it administers a redistributive program by making a lump-sum transfer of its seigniorage revenues, $q_t$, to the individual in the form of nominal balances.

In addition to the budget constraint to be specified later, the representative agent also faces a cash-in-advance constraint (CIA) towards its purchases of the nonstorable consumption good. Namely, such purchases are financed directly by the beginning-of-period cash balances, which are the sum of balances from the period, $m_t$, and transfers from the government, $\tau_t$.

$$p_t c_t \leq m_t + \tau_t,$$

where $p_t$ is the price level in period $t$.

Since a key feature of our model lies in its organizational structure which involves monopolistic competition and increasing returns, the individual income comes from not only physical capital, $k_t$, and labor effort, $h_t$, but also ownership of firms in the form of positive profits, $\pi_t$. Income can be spent on either consumption, capital investment ($i_t$ in real terms), or the accumulation of nominal cash balances to take into next period ($m_{t+1}$). Let the rate of return (net of depreciation) on capital be $r_t$, and the wage rate be $w_t$. Then the budget constraint is

$$c_t + i_t + m_{t+1}/p_t \leq w_t h_t + r_t k_t + \pi_t + (m_t + \tau_t)/p_t,$$

where investment and capital satisfy the following law of motion:

$$k_{t+1} = (1 - \delta)k_t + i_t.$$  

The constant parameter $\delta \in [0,1]$ is the depreciation rate of physical capital.

On the production side, following Benhabib and Farmer (1994), there are a continuum of monopolistically competitive intermediate goods producers, indexed by $j \in (0,1)$, who face increasing returns technologies. Final output, produced in a competitive sector, is given by

$$Y = \left[ \int_0^1 Y(j)^{1/\lambda} \, dj \right]^{1/\lambda},$$

where $\lambda \in (0,1)$ measures the degree of monopoly power in the markets for intermediate products. The smaller the value of $\lambda$, the higher the markup, and the higher the monopoly profits. Throughout this paper, we shall use capital
In Section 4, our calibration of the model takes the same parameter values from Farmer and Guo (1994). Specifically, we take \( \lambda = 0.58 \), \( a = 0.23 \), \( b = 0.7 \), and hence \( a + b < 1 \) is satisfied; moreover, these numbers imply that \( \alpha = 0.4 \), \( \beta = 1.21 \), and so \( \alpha + \beta > 1 \).

Letters to denote (per-capita) economy-wide variables, and lower-case letters to denote (per-capita) individual variables. For ease of exposition, we deliberately keep them distinct from each other, although in equilibrium they equalize. The production function for intermediate good \( j \) is assumed to possess the Cobb–Douglas form,

\[
Y_t(j) = Z_t(j)K_t(j)\alpha H_t(j)\beta, \quad \alpha, \beta > 0, \quad \alpha + \beta > 1,
\]

where \( Z_t(j) \) is a productivity shock. To keep things simple, we follow Benhabib and Farmer (1994) by assuming that the same technology is used to produce each intermediate good. Under this symmetry assumption, we have \( Z_t(j) = Z_t \), \( Y_t(j) = Y_t \), \( K_t(j) = K_t \), and \( H_t(j) = H_t \). Thus, the aggregate production function can be expressed as

\[
Y_t = Z_tK_t^\alpha H_t^\beta.
\]

The productivity shock \( Z_t \) evolves according to

\[
Z_t = Z_{t-1}^{\theta_1} u_t, \quad \theta_1 \in (0,1), \quad Z_0 \text{ is given}.
\]

The autocorrelation of the productivity shock, \( Z_t \), is intended to capture the fact that U.S. GNP requires at least a second-order representation suggested by the data. In two of the models below, the innovation, \( u_t \), is specified as an i.i.d. random variable which follows a lognormal distribution with mean zero, while in the other model \( Z \) is specified as a constant equal to unity.

Under the hypothesis that factor markets are competitive, standard first-order conditions of the representative firm lead to

\[
r_t = aY_t/K_t, \quad w_t = bY_t/H_t, \quad a = \lambda \alpha, \quad b = \lambda \beta,
\]

where \( a \) and \( b \) are the capital and labor shares in national income, respectively. Notice that in this monopolistically competitive model, the sum of factor shares of national income is less than one, \( a + b < 1 \), thereby allowing for the presence of positive profits. By contrast, in a model with competitive input markets and constant returns, \( \alpha + \beta = 1, \alpha = a, \) and \( \beta = b \).

Finally, the budget constraint for the government is

\[
\tau_t = (g_t - 1)M_t,
\]

where \( M_t \) is per-capita money stock (exogenous). Its gross growth rate, \( g_t \), evolves according to

\[
g_t = g_t^{\theta_2} v_t, \quad \theta_2 \in (0,1), \quad g_0 \text{ is given},
\]

\[5\] In Section 4, our calibration of the model takes the same parameter values from Farmer and Guo (1994). Specifically, we take \( \lambda = 0.58 \), \( a = 0.23 \), \( b = 0.7 \), and hence \( a + b < 1 \) is satisfied; moreover, these numbers imply that \( \alpha = 0.4, \beta = 1.21 \), and so \( \alpha + \beta > 1 \).
where \( v_t \) is a random shock, which follows a lognormal distribution with mean zero. For the model economy, we also consider a special case in which \( g_t \) is a constant. This then is qualitatively equivalent to the standard RBC model without monetary shocks.

2.2. Market equilibrium

In the model laid out above, it is observed that some variables display growth in equilibrium; e.g., with a growing money supply, nominal prices are clearly not stationary. To ensure the existence of stationary market equilibria, we need to transform them into variables that are scaled appropriately to eliminate growth. To this end, denote \( m_M = m/M \), \( p_M = p/M \), and \( \tau_M = \tau/M \), where the superscript time subscripts have been dropped; also denote a prime (’) as the next-period value, and the aggregate state as \( S = (Z, g, K) \). Then prices, distributed profits and government transfers can presumably be expressed as functions of the aggregate state; that is, \( p_M = p_M(S) \), \( r = r(S) \), \( w = w(S) \), \( \pi = \pi(S) \), and \( \tau_M = \tau_M(S) \). As a result, the problem faced by the representative agent is to choose consumption \( (c, k) \), labor effort \( (h) \), stock of physical capital \( (k') \), and cash balances \( (m_M) \), so as to solve the following dynamic programming problem:

\[
V(k, m_M, S) = \max_{c, k', m_M} \{ U(c, h) + \rho E[V(k', m_M', S')(k, m_M, S)] \}, \tag{12}
\]

\[
\text{s.t.} \quad c + k' + g m_M'/p_M(S) \leq w(S)h + [1 - \delta + r(S)]k + \pi(S) + [m_M + \tau_M(S)]/p_M(S), \tag{13a}
\]

\[
p_M(S)c \leq m_M + \tau_M(S), \tag{13b}
\]

\[
S' = S(Z(S, u'), g(S, v'), K(S)). \tag{13c}
\]

**Definition.** A stationary market equilibrium consists of policy functions \( c(s), h(s), k(s), m_M'(s), K(s), \) and \( H(s) \), where \( s = (k, m_M, S) \); pricing functions \( p_M(S), r(S), \) and \( w(S) \); a profit function \( \pi(S) \); and a transfer function \( \tau_M(S) \) such that (i) the functions \( V, K, H \), and \( p_M \) satisfy (12) and \( c, h, k, \) and \( m_M' \) are the associated policy functions; and (ii) the money market, the labor market, the capital market, and the goods market all clear: \( m_M = m_M' = 1, h = H, k = K, \) and

\[
c + k' = Y + (1 - \delta)k. \tag{14}
\]

In what follows, the attention will be focused on circumstances under which the CIA constraint always holds with equality. As is well known, a sufficient condition for this constraint to be binding is that the gross growth rate of
money, \( g_t \), always exceeds the discount factor, \( \rho \).\(^6\) Consequently, the conditions characterizing this dynamic optimum include the market-clearing conditions (notably (14)) and

\[
U_h(c, h)/w = \rho E[(1 - \delta + r')U_h(c', h')/w'], \tag{15a}
\]

\[
gU_h(c, h)/(p_M w) = \rho E[U_c(c', h')/p_M'], \tag{15b}
\]

\[
k' = Zk^2h^\beta + (1 - \delta)k - g/p_M. \tag{15c}
\]

Notice that Eq. (15c) is the goods market-clearing condition or resource constraint expressed in per-capita form when the individual CIA constraint is binding.

3. Dynamics and expectational shocks

In this section, we study the transitional dynamics implied by the model by taking a linear approximation of the system around an initial stationary equilibrium. To accomplish this, it remains to specify the instantaneous utility function. As in related studies, it is parameterized as

\[
U(c, h) = \ln c - Bh, \tag{16}
\]

where \( B \) is a constant parameter. The use of such a functional form can be justified by the following two reasons. First, this form of indivisible labor is standard in the literature, see, e.g., Hansen (1985), Rogerson (1988) and Cooley and Hansen (1989) among others. With the form like (16), we can compare the implied welfare costs of inflation in our model economy of monopolistic competition and strong increasing returns with those in existing models of competitive markets and constant returns. Second, the informational requirements are substantially reduced by this simplification. Namely, Eq. (15b) can be eventually simplified to involve only one unknown, \( p_M \); as a result, solving for \( p_M \) and substituting into Eq. (15c) will leave the dynamic system with only two difference equations in \( k \) and \( h \).

\[
Z^{-1}k^{-\gamma}h^{1-\beta} = E[\rho a(k)^{-1}h' + \rho(1 - \delta)(Z')^{-1}(k')^{-\gamma}(h')^{1-\beta}], \tag{17}
\]

\[
k' = Zk^2h^\beta + (1 - \delta)k - (\rho b/B)Zk^2h^\beta - 1 E(1/g'). \tag{18}
\]

\(^6\) In Section 4 on empirical results, this is equivalent to assuming that the annual inflation rate is equal to or higher than \(-4\%\), which is a reasonable assumption.
By setting the steady-state values $Z^* = 1$ and $g^* = g$ (a constant), the steady-state values of $k^*$ and $h^*$ can be readily obtained:

$$h^* = \frac{\rho b}{gB} \frac{1 - \rho(1 - \delta)}{1 - \rho(1 - \delta) - \alpha \rho \delta}, \quad k^* = \left[ \frac{a \rho}{1 - \rho(1 - \delta)} (h^*)^\delta \right]^{1/(1 - \delta)}. \tag{19}$$

An additional equation specifies how the steady-state values of $c^*$ and $p_{Ht}^*$ are interrelated:

$$c^* = g/p_{Ht}^* = (\rho b/gB)(k^*)^a(h^*)^{-\delta}. \tag{20}$$

Next the dynamics of our stochastic economy are described as follows. Let a circumflex denote the deviation of a variable from its steady-state value expressed as a percentage of that value, e.g., $\hat{k} = (k - k^*)/k^*$. First, we take a first-order Taylor series approximation to Eqs. (8), (11), (17) and (18) around the steady state. We then carry out some straightforward algebraic manipulation on the linearized equations by exploiting the cross-equation restrictions. The resulting solution of this system can be written in the form

$$
\begin{bmatrix}
\hat{k} \\
\hat{h} \\
\hat{Z} \\
\hat{g}
\end{bmatrix}
= A
\begin{bmatrix}
k \\
h \\
Z \\
g
\end{bmatrix}
+ D
\begin{bmatrix}
\hat{u} \\
\hat{v}
\end{bmatrix},
\tag{21}
$$

where $\hat{e}_h \equiv h' - E(h')$ and $A$ and $D$ are, respectively, $4 \times 4$ and $4 \times 3$ matrices defined in a technical appendix available upon request. The term $\hat{e}_h$ represents disturbances to labor effort that have zero-conditional mean at time $t$. In the remainder of this paper, $\hat{e}_h$ is interpreted as a self-fulfilling belief.

It is clear that the dynamic characteristics of the decision rules (21) around the steady state are governed by the transition matrix, $A$. In standard RBC frameworks, this matrix possesses four real eigenvalues, one of which is greater than unity in absolute value, while the other three are smaller than one in absolute values. In this case, the system (21) implies a saddle point equilibrium with a unique convergent path. In this paper, however, if the returns to scale are strong, then all roots of the matrix $A$ lie inside the unit circle. Therefore, similar to Farmer and Guo (1994) who introduce increasing returns to a real economy, our model generates an important implication: its steady state has a sink with multiple paths converging towards it, and hence the belief shock to the labor effort equation, $\hat{e}_h$, inevitably affects its dynamic path and may be an important

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7 More precisely, given appropriate parameterization, we can show that in our multidimensional system, the two roots associated with capital and hours worked are complex conjugates. See the discussion in the following section.
source of business cycle fluctuations. In the extreme case of no fundamental shocks, i.e., \( \dot{u} = \dot{v} = 0 \), strong increasing returns can induce indeterminacy of the equilibrium, leaving the belief shocks a sole source of fluctuations that can magnify the effects of other shocks.\(^8\) Therefore, we refer to this model as the ‘sunspot’ model.

To gain further insights about the effects of introducing strong increasing returns, we calibrate and simulate the sunspot model and also two compatible versions of the above system, both with the dynamic property that all roots of the transition matrix, \( A \), are real, with one larger than one in absolute value and the others less than one in absolute values. The first one is a standard monetary RBC model with competitive markets and a technology that satisfies constant returns to scale, whereas the second, although with monopolistically competitive markets, exhibits mild increasing returns to scale and saddle point dynamics.\(^9\) These two economies are termed as the ‘competitive’ and ‘saddle’ economies, respectively. The difference between this second economy and ours is that our economy displays an indeterminate approach to the steady state, which, as recognized by Farmer and Guo (1994), implies that the dynamics of our linearized model no longer provide enough restrictions to uniquely determine a rational expectations equilibrium in terms of fundamentals.

For these two models, using the saddle-path property as a restriction, the decision rules (21) can be simplified as follows:

\[
\begin{align*}
\dot{K} &= b_{11} \dot{K} + b_{12} \dot{z} + b_{13} \dot{g}, \\
\dot{z} &= \theta_1 \dot{z} + \dot{u}, \\
\dot{g} &= \theta_2 \dot{g} + \dot{v}, \\
 b_{21} \dot{K} + b_{22} \dot{h} + b_{23} \dot{z} + b_{24} \dot{g} &= 0,
\end{align*}
\]

where (22d) is the equation of the saddle. This equation and the constant coefficients, \( b \)'s, are derived in a technical appendix which is available from the authors upon request. Notably, unlike that in (21), expectational errors, \( \ddot{\epsilon}_h \) vanish in (22), leaving the system to be driven exclusively by fundamental shocks. Interestingly, (22) implies that the data can be described as a third-order system, while (21) suggests that they can be described as a fourth-order system.

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\(^8\) As shown in Benhabib and Farmer (1994), the key feature which causes a change in the stability of the steady state is the fact that strong increasing returns imply that the labor demand curve may slope up and is steeper than the labor supply curve. It can be readily verified that the slopes of the demand and supply curves are \( \beta - 1 \) and 0, respectively. So, the necessary condition for indeterminacy requires \( \beta > 1 \).

\(^9\) An alternative model would be to assume price-taking behavior but incorporate increasing returns by externalities, similar to the one studied in Baxter and King (1991) without money.
4. Model calibration and diagnoses

In this Section, we provide some simple diagnoses of our sunspot model. As in most recent studies of this type, the model will be evaluated on how well it accounts for the observed cyclical properties of the key U.S. macroeconomic variables. To further investigate the model implications, we also compare it with two stylized models, namely, the ‘competitive’ and the ‘saddle’ economies outlined in the preceding section.

4.1. Calibration

The parameter values of the model economies are determined by, either using micro evidence or matching steady-state values of certain variables with their long-run averages observed in the data. The preference parameters are set to \( \rho = 0.99 \) and \( B = 2.86 \), the same as in Cooley and Hansen (1989) for a comparative study of the two models. The quarterly depreciation rate for the capital stock, \( \delta \), is set to 0.025, indicating an annual rate of 10\%, as in Kydland and Prescott (1982) and their followers. For the scale parameters, in the competitive economy with constant returns, the capital and labor shares of national income are chosen to be \( \alpha = 0.36 \) and \( \beta = 0.64 \), respectively. In the other two economies with noncompetitive markets and increasing returns, we follow Farmer and Guo (1994) in specifying these parameter values. Specifically, in the saddle economy with monopolistic competition and moderate increasing returns, the degree of monopoly power, \( \lambda \), is taken to be 0.7, and \( \alpha = 0.43 \) and \( \beta = 0.9 \); in the sunspot economy, the respective values are \( \lambda = 0.58 \), \( \alpha = 0.4 \), and \( \beta = 1.21 \).

Other parameter values are specified as follows: The mean growth rate of money supply is set to 1.5\%, i.e., \( \bar{\gamma} = 1.015 \). Estimation of the AR(1) process (11) using the growth rate of the U.S. money supply \( M1 \) yields \( \theta_2 = 0.60 \) and \( \sigma_2 = 0.0081 \). Similarly, for the first two economies, aggregate fluctuations also stem from productivity shocks, as governed by Eq. (8). As in related studies, we set the first-lag autocorrelation coefficient, \( \theta_1 \), to 0.95, whilst the standard deviation of technological innovation, \( \sigma_u \), is parameterized so as to match the standard deviation of output generated by the model economies with that computed from the actual U.S. data. The latter leads to \( \sigma_u = 0.007 \) for the competitive economy and \( \sigma_u = 0.0046 \) for the saddle economy.

As for the sunspot economy, we consider two variants: one with a combination of money and expectational shocks and the other with merely expectational shocks. Such a differentiation enables us to study the behavior of nominal variables in the former. It should be noted that in both circumstances economic fluctuations are largely or entirely propagated by shocks to self-fulfilling beliefs, which are reflected in the work effort equation (see Eq. (21)). For analytical purposes, the expectational shock term \( \hat{\varepsilon}_h = h' - E(h') \) is assumed to follow
a white-noise process with a constant standard deviation $\sigma_{\epsilon}$. In the absence of productivity shocks, this parameter is selected so that the standard deviation of output generated by the artificial economy equals its empirical counterpart. As a result, $\sigma_{\epsilon_{1}} = 0.0103$ is taken for the first variant, while $\sigma_{\epsilon_{2}} = 0.0107$ is taken for the second one.

The above choice of parameter values implies that for the first two economies, all eigenvalues of the transition matrix, $A$ in Eq. (21), are real numbers. Furthermore, the second root is greater than unity in absolute value, while the other three are smaller than unity. It is therefore known that each of these two economies can be characterized by a saddle-path equilibrium, suggesting that the disturbance term to the work effort equation can be exclusively determined from the fundamentals of the economy by the cross-equation restriction (22d) that places the economy on the stable branch of the saddle. By contrast, for the sunspot economy, increasing returns are sufficiently strong that all eigenvalues of $A$ lie inside the unit circle. In particular, the two roots associated with capital and work effort are complex conjugates. As discussed in the preceding section, such complex roots are completely capable of generating oscillations in the economy even without serially correlated productivity shocks. Note that in this economy shocks to work effort are not only determined by economic fundamentals, but can be driven by self-fulfilling beliefs as well.

4.2. Model diagnoses

Column (1) of Table 1 presents the standard deviations of major U.S. macroeconomic series and their correlations with output (in parentheses). Our data set includes 151 quarterly observations which span the period from 1954.I to 1991.III, all seasonally adjusted. The data on real variables (output, consumption, capital stock, investment and hour measures) coincide with those in Farmer and Guo (1994) who kindly provided their data. The series of nominal variables ($M_{1}$ money stock, consumer price index and implicit GDP deflator) are taken from the Citicorp’s Citibase data bank. All statistics shown in Table 1 have been computed using data that are first logged and then detrended by the Hodrick–Prescott filter.

In each model economy, 151 artificial observations are simulated using respective parameter values chosen above. For the first two economies which possess a saddle-path equilibrium, innovations to productivity and monetary growth are randomly drawn from i.i.d. normal distributions $N(0,\sigma_{\zeta}^{2})$ and $N(0,\sigma_{\psi}^{2})$, respectively; these sequences are then utilized to construct capital and labor effort series according to decision rules (22). For the first version of the sunspot economy, we simulate monetary innovations and expectational shocks from i.i.d. distributions $N(0,\sigma_{\zeta}^{2})$ and $N(0,\sigma_{\psi}^{2})$, respectively, whereas for the
Table 1
Standard deviations and correlations with output for the U.S. and simulated economies*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.74</td>
<td>1.73</td>
<td>1.73</td>
<td>1.73</td>
<td>1.73</td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td>(1.00)</td>
<td>(1.00)</td>
<td>(1.00)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.86</td>
<td>0.68</td>
<td>0.72</td>
<td>0.63</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>(0.77)</td>
<td>(0.64)</td>
<td>(0.75)</td>
<td>(0.46)</td>
<td>(0.79)</td>
</tr>
<tr>
<td>Capital stock</td>
<td>0.47</td>
<td>0.48</td>
<td>0.53</td>
<td>0.76</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.05)</td>
<td>(0.06)</td>
<td>(0.03)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Investment</td>
<td>7.78</td>
<td>5.56</td>
<td>6.27</td>
<td>9.37</td>
<td>9.02</td>
</tr>
<tr>
<td></td>
<td>(0.90)</td>
<td>(0.96)</td>
<td>(0.95)</td>
<td>(0.95)</td>
<td>(0.99)</td>
</tr>
<tr>
<td>Hours 1</td>
<td>1.51</td>
<td>1.32</td>
<td>1.27</td>
<td>1.43</td>
<td>1.44</td>
</tr>
<tr>
<td></td>
<td>(0.86)</td>
<td>(0.98)</td>
<td>(0.97)</td>
<td>(0.98)</td>
<td>(0.98)</td>
</tr>
<tr>
<td>Hours 2</td>
<td>1.68</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.88)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity 1</td>
<td>0.88</td>
<td>0.50</td>
<td>0.54</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>(0.49)</td>
<td>(0.86)</td>
<td>(0.86)</td>
<td>(0.77)</td>
<td>(0.79)</td>
</tr>
<tr>
<td>Productivity 2</td>
<td>0.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price (CPI)</td>
<td>1.34</td>
<td>2.08</td>
<td>2.11</td>
<td>2.15</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>(−0.57)</td>
<td>(−0.20)</td>
<td>(−0.28)</td>
<td>(−0.31)</td>
<td>(−0.79)</td>
</tr>
<tr>
<td>Price (Def)</td>
<td>0.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−0.44)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Notes: (1) Data sources: the U.S. time series of output, consumption, capital stock, investment, hours 1, hours 2, productivity 1 and productivity 2 are taken from Farmer and Guo (1994). The price series are from Citicorp’s Citibase data bank. (2) Numbers in the first lines are standard deviations, while those in the second lines (in parentheses) are correlations with output.

Second version, only the expectational shocks are drawn. The decision rules at work now are the set of (21). Each economy is simulated 100 times and the averages of the moments are reported in Columns (2)–(5) of Table 1, respectively.

We then perform two diagnoses: volatility and comovement. The results are presented in Columns (2)–(5) of Table 1. Due to space limitation, the detailed discussion is omitted here. In a word, these exercises demonstrate that the sunspot model matches the volatilities and comovements of economic aggregates with about the same degree of precision as the standard monetary RBC models. This success gives much credence to the model for the measurement of welfare costs of inflation in the following sections.
5. Welfare costs of inflation

In this section, we compute the welfare costs of inflation under our sunspot economy and compare these cost measures with those implied under the competitive and saddle economies for various growth rates of money supply. We follow Cooley and Hansen (1989) in basing the estimates on the steady-state properties of the economy with perfect foresight.

We derive the cost measure in conventional fashion. For each model economy, the welfare cost under an alternative money growth rate is expressed as a compensation in consumption in such a way that individuals would be as well off as if there were no CIA constraint. This measure of the welfare cost is appropriate for economies with competitive markets and constant returns, as the equilibrium with non-binding CIA corresponds to a Pareto-optimal allocation by the first welfare theorem. For the other two economies, in which market imperfection is a central ingredient, however, it creates a minor conceptual difficulty, in that the corresponding market equilibrium is no longer Pareto optimal, even though the CIA constraint does not bind. Nevertheless, such a case remains the best outcome for economic agents since they attain the highest utility compared to all other possible rates of money growth. We shall therefore continue to refer to the equilibrium with the non-binding CIA constraint as the ‘optimal’ solution for all economies.

As noted in Section 3, when the growth rate of money falls below the discount factor ($\bar{g} \leq \rho$), the CIA constraint is no longer binding. Let $c^*_H$ and $h^*_H$ denote the steady-state values of consumption and hours worked under this optimal growth rate, and $c^A$ and $h^A$ denote the respective steady-state values under an alternative money growth rate. The welfare cost, denoted by $\Delta c$, is defined as follows:

$$U(c^H, h^H) = U(c^A + \Delta c, h^A).$$  

(23)

Under the assumption of utility function (16), the welfare cost $\Delta c$ in (23) can be expressed as the percentages of steady-state consumption and output, respectively:

$$\frac{\Delta c}{c^A} = \left(\frac{\bar{g}^\Lambda}{\rho}\right)^{\beta(1-\alpha)} \exp \left[ \phi \left( \frac{\rho}{\bar{g}^\Lambda} - 1 \right) \right] - 1,$$  

(24)

$$\frac{\Delta c}{Y^A} = \frac{\lambda \beta}{\phi} \left(1 - \frac{\Delta c}{c^A}\right) = \frac{1 - \rho(1 - \delta) - \lambda z \rho \delta \left(\frac{\Delta c}{c^A}\right)}{1 - \rho(1 - \delta)} \left(\frac{\Delta c}{c^A}\right),$$  

(25)

where $\phi = \lambda \beta [1 - \rho(1 - \delta)]/[1 - \rho(1 - \delta) - \lambda z \rho \delta]$ is a constant parameter.

To make our results comparable to those in Cooley and Hansen (1989), Table 2 presents the welfare costs as well as the steady-state values of some key variables for all three model economies under five alternative money supply...
Table 2
Steady state and welfare costs of inflation with quarterly CIA constraint in consumption

<table>
<thead>
<tr>
<th>Annual inflation rate</th>
<th>Column (1)</th>
<th>Column (2)</th>
<th>Column (3)</th>
<th>Column (4)</th>
<th>Column (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−4% ( g = \rho )</td>
<td>0.0% ( g = 1.0 )</td>
<td>10% ( g = 1.024 )</td>
<td>100% ( g = 1.19 )</td>
<td>300% ( g = 1.41 )</td>
<td></td>
</tr>
</tbody>
</table>

### The competitive model

#### Steady state

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Capital stock</th>
<th>Investment</th>
<th>Hours</th>
<th>Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>−4%</td>
<td>1.115</td>
<td>0.829</td>
<td>11.432</td>
<td>0.286</td>
<td>0.301</td>
<td>3.704</td>
</tr>
<tr>
<td>0.0%</td>
<td>1.104</td>
<td>0.821</td>
<td>11.318</td>
<td>0.283</td>
<td>0.298</td>
<td>3.704</td>
</tr>
<tr>
<td>10%</td>
<td>1.077</td>
<td>0.801</td>
<td>11.052</td>
<td>0.276</td>
<td>0.290</td>
<td>3.704</td>
</tr>
<tr>
<td>100%</td>
<td>0.927</td>
<td>0.689</td>
<td>9.511</td>
<td>0.237</td>
<td>0.250</td>
<td>3.704</td>
</tr>
<tr>
<td>300%</td>
<td>0.782</td>
<td>0.582</td>
<td>8.027</td>
<td>0.200</td>
<td>0.211</td>
<td>3.704</td>
</tr>
</tbody>
</table>

#### Welfare costs

<table>
<thead>
<tr>
<th></th>
<th>(( \Delta c/c^A )) \times 100</th>
<th>(( \Delta c/Y^A )) \times 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>−4%</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.0%</td>
<td>0.144</td>
<td>0.107</td>
</tr>
<tr>
<td>10%</td>
<td>0.520</td>
<td>0.387</td>
</tr>
<tr>
<td>100%</td>
<td>4.013</td>
<td>2.984</td>
</tr>
<tr>
<td>300%</td>
<td>10.215</td>
<td>7.596</td>
</tr>
</tbody>
</table>

### The saddle model

#### Steady state

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Capital stock</th>
<th>Investment</th>
<th>Hours</th>
<th>Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>−4%</td>
<td>0.679</td>
<td>0.533</td>
<td>5.825</td>
<td>0.145</td>
<td>0.280</td>
<td>2.422</td>
</tr>
<tr>
<td>0.0%</td>
<td>0.668</td>
<td>5.525</td>
<td>5.733</td>
<td>0.143</td>
<td>0.277</td>
<td>2.408</td>
</tr>
<tr>
<td>10%</td>
<td>0.644</td>
<td>0.506</td>
<td>5.522</td>
<td>0.138</td>
<td>0.271</td>
<td>2.375</td>
</tr>
<tr>
<td>100%</td>
<td>0.508</td>
<td>0.399</td>
<td>4.356</td>
<td>0.108</td>
<td>0.233</td>
<td>2.177</td>
</tr>
<tr>
<td>300%</td>
<td>0.388</td>
<td>0.305</td>
<td>3.332</td>
<td>0.083</td>
<td>0.196</td>
<td>1.974</td>
</tr>
</tbody>
</table>

#### Welfare costs

<table>
<thead>
<tr>
<th></th>
<th>(( \Delta c/c^A )) \times 100</th>
<th>(( \Delta c/Y^A )) \times 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>−4%</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.0%</td>
<td>0.788</td>
<td>0.619</td>
</tr>
<tr>
<td>10%</td>
<td>2.704</td>
<td>2.125</td>
</tr>
<tr>
<td>100%</td>
<td>16.854</td>
<td>13.240</td>
</tr>
<tr>
<td>300%</td>
<td>37.645</td>
<td>29.574</td>
</tr>
</tbody>
</table>

### The sunspot model

#### Steady state

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Capital stock</th>
<th>Investment</th>
<th>Hours</th>
<th>Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>−4%</td>
<td>0.298</td>
<td>0.248</td>
<td>1.970</td>
<td>0.049</td>
<td>0.294</td>
<td>1.014</td>
</tr>
<tr>
<td>0.0%</td>
<td>0.292</td>
<td>0.243</td>
<td>1.931</td>
<td>0.048</td>
<td>0.291</td>
<td>1.004</td>
</tr>
<tr>
<td>10%</td>
<td>0.278</td>
<td>0.232</td>
<td>1.841</td>
<td>0.046</td>
<td>0.284</td>
<td>0.980</td>
</tr>
<tr>
<td>100%</td>
<td>0.205</td>
<td>0.171</td>
<td>1.359</td>
<td>0.034</td>
<td>0.244</td>
<td>0.841</td>
</tr>
<tr>
<td>300%</td>
<td>0.146</td>
<td>0.122</td>
<td>0.965</td>
<td>0.024</td>
<td>0.206</td>
<td>0.708</td>
</tr>
</tbody>
</table>

#### Welfare costs

<table>
<thead>
<tr>
<th></th>
<th>(( \Delta c/c^A )) \times 100</th>
<th>(( \Delta c/Y^A )) \times 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>−4%</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.0%</td>
<td>1.193</td>
<td>0.996</td>
</tr>
<tr>
<td>10%</td>
<td>4.100</td>
<td>3.422</td>
</tr>
<tr>
<td>100%</td>
<td>25.831</td>
<td>21.563</td>
</tr>
<tr>
<td>300%</td>
<td>58.843</td>
<td>49.120</td>
</tr>
</tbody>
</table>

rules, where the period over which the agent is constrained to hold cash is one quarter. Column (1) corresponds to the optimal allocation in which the CIA constraint is not binding, or equivalently the economy experiences an inflation rate of −4%.
Two striking results emerge from Table 2: first, the welfare costs of various money growth rates in either of the two increasing returns economies are considerably higher than those in the Cooley–Hansen (1989) economy; and second, the stronger the increasing returns, the higher the welfare costs. For example, a 10% money growth results in welfare costs of 2.125 and 3.422% in the saddle and sunspot economies, respectively, as opposed to 0.387% in the Cooley–Hansen economy. The significantly higher welfare costs in increasing returns economies are consistent across all money growth rates.

To develop a better understanding of the welfare costs of inflation under the sunspot economy, we consider several variations of the model parameters.

First, for the baseline values of the model parameters, we compute the welfare costs for the annual inflation rates ranging from the optimal level of 4–100%, and plot the results in Panel A of Fig. 1. It can be clearly seen that as the inflation rate increases, both measures of the welfare cost rise monotonically. For example, at the 5% inflation rate, $\Delta c/c^A = 2.7\%$, and $\Delta c/Y^A = 2.2\%$. However as the inflation rate goes up to 10%, $\Delta c/c^A$ increases to 4.1% and $\Delta c/Y^A$ to 3.4%.

One central element of the sunspot economy, which makes it different from the traditional neo-classical models, is that the production technology in the intermediate goods sector is subject to strong increasing returns to scale. Our second experiment is to investigate how significant returns to scale are in affecting the welfare cost of inflation. Notice that the production technology (7) is described by two parameters $\alpha$ and $\beta$. We fix the value of capital elasticity $\alpha$ at its benchmark value of 0.4, while allowing the value of labor elasticity $\beta$ to change from 0.6 which corresponds to the constant returns to scale case to 1.21 which is the benchmark value used in computing the results in Table 2. All other model parameters remain unchanged and the annual inflation rate is assumed to be 10%. Panel B of Fig. 1 shows unambiguously the positive relationship between the welfare cost and the value of the scale parameter $\beta$. In particular, when the economy is subject to constant returns to scale ($\beta = 0.6$), the welfare cost for a 10% annual inflation is $\Delta c/c^A = 2.0\%$, and $\Delta c/Y^A = 1.7\%$. However,
Fig. 1. Welfare costs of inflation: (a) welfare costs of alternative inflation rates, (b) welfare costs of 10% inflation with alternative $\beta$’s, (c) welfare costs of 10% inflation with alternative $\lambda$’s.

these cost measures rise steadily as $\beta$ increases and when $\beta = 1.21$, $\Delta c/c^A = 4.1\%$, and $\Delta c/Y^A = 3.4\%$. Similarly, we have examined the effects of increasing returns economy on the welfare cost by fixing the labor elasticity $\beta$, while varying the value of capital elasticity $\lambda$. The results (not reported) establish the same pattern that there is a positive relationship between the welfare cost and the value of $\lambda$.

Another essential ingredient of our model is the assumption of imperfect competition among intermediate goods producers. In Eq. (5), the parameter $\lambda$ measures the degree of monopoly power in the intermediate goods markets.
Therefore, our last experiment studies the relationship between the welfare cost of inflation and the value of $\lambda$. We calculate the cost of a 10% inflation for various values of $\lambda$ ranging from the baseline value, 0.58, to the perfect competition case, 1.0, while maintaining all other model parameters at the original level. Results presented in Panel C of Fig. 1 plainly indicate that both measures of the welfare cost monotonically decrease as competition among firms intensifies. In particular, $\Delta c/c^A$ and $\Delta c/Y^A$ obtain much lower values, 1.0 and 0.7%, respectively, as the economy approaches perfect competition, $\lambda = 1.0$.

The above analysis indicates that for a given level of inflation, the higher the returns to scale and/or the less competitive the market, the higher the welfare cost. To gain some intuition about these results, consider, at the steady state, the wedge between the marginal utility of consumption, $U_c$, and the shadow price of wealth, $U_h/w$. Using (9) and (20), the equilibrium real wage rate can be expressed as

$$ w = gBc^*/\rho, \quad (26) $$

and therefore the wedge between the marginal utility of consumption and the shadow price of wealth can be written as

$$ U_c + U_h/w = (1 - \rho/g) / c^*. \quad (27) $$

Eq. (27) says that when the money growth rate, $g$, is higher than the discount factor $\rho$ (or equivalently when the CIA constraint is binding), there exists a discrepancy between the marginal utility of consumption and the shadow price of wealth, thereby resulting in a welfare loss. Furthermore, this welfare loss is inversely related to the steady-state level of consumption. How do the returns to scale and the degree of competition affect the steady-state consumption? Straightforward derivation using (19) and (20) yields

$$ \frac{\partial (\ln c^*)}{\partial \beta} = \frac{1}{1 - x} \left[ 1 + \ln h^* \right] $$

$$ = \frac{1}{1 - x} \left[ 1 + \ln \left( \frac{\beta \rho \delta}{gB} \frac{1 - \rho(1 - \delta)}{1 - \rho(1 - \delta) - x \rho \delta \lambda} \right) \right]. \quad (28) $$

$$ \frac{\partial (\ln c^*)}{\partial \lambda} = \frac{1}{1 - x} \left[ \frac{x + \beta}{\lambda} + \frac{x \rho \delta (x + \beta - 1)}{1 - \rho(1 - \delta) - x \rho \delta \lambda} \right]. \quad (29) $$

Clearly (29) is unambiguously positive. As for Eq. (28), we verify numerically that $\ln h^* < -1$ for the ranges of parameter values chosen for all three model economies, and therefore (28) is negative.

These conditions demonstrate that for a given level of inflation, higher returns to scale (higher $\beta$) and/or higher monopoly power (lower $\lambda$) depress the steady-state consumption, which in turn raises the marginal utility of consumption.
While the shadow price of wealth increases through a lower real wage rate (see (26)), it does so more slowly than the increase in the marginal utility of consumption when the CIA constraint is binding. Therefore, the wedge defined in (27) increases, which results in a higher welfare cost.

Our results on the effects of returns to scale can also be intuitively understood through the interest elasticity of money demand. To this end, we calculate the interest elasticities of money demand in steady states for all three economies. It is known from Section 3 that the real (gross) rate of interest, $1 + r - \delta$, equals $1/\rho$, (see (15a)); and that the (gross) rate of inflation equals the rate of money growth, $g$. These imply that the nominal (gross) rate of interest is simply $R \equiv g/\rho$. From Eqs. (19), (20), and the definition of $R$, it is easy to obtain the elasticity measure, $e$:

$$
e \equiv R/(M^*/P^*)\partial(M^*/P^*)/\partial R = -2 - (\alpha + \beta - 1)/(1 - \alpha). \quad (30)$$

Clearly, the second term in Eq. (30) vanishes in the Cooley–Hansen economy due to constant returns, while it rises in absolute value as increasing returns become stronger. The elasticities in the competitive, saddle, and sunspot economies are $-2.00$, $-2.58$, and $-3.02$, respectively. Thus, the ranking of the interest elasticities confirms the ranking of the welfare costs, as in Gillman (1993). This result accords reasonably well with intuition. Since money demand is more interest-elastic in the sunspot economy than in the Cooley–Hansen economy, a given money growth induces larger reductions in money demand and hence consumption (via CIA constraint) for the former economy. This larger drop in consumption then gives rise to a higher cost.

6. Other model specifications

The preceding section documents the relatively higher welfare cost of inflation under our sunspot economy. A natural question arises as to whether these results are robust to accommodate other model specifications, to which the attention now turns in this section.

We consider three alternative specifications:

- a CES utility function where consumption and leisure are non-separable;
- a more general CIA constraint in which all purchases must be made with currency;
- a transactions constraint which relates household holdings of real money balances and the fraction of the time devoted to transacting to the spending flow that the household carries out.

---

12 This can be directly verified from the definition of $p_M = p/M$. 
We experiment with other values of these parameters and find the results insensitive to the choices of parameters.

We provide the utility and money demand functions under each specification. The detailed derivations for the steady-state values of key variables as well as the closed-form solutions for the welfare function are given in a technical appendix which is available from the authors upon request.

A CES utility function, which is also adopted by Gomme (1993), is given by

\[ U(c, h) = \left[ c^{\omega}(1 - h)^{1 - \omega}\right]^{1/(1 - \sigma)}. \]  

(31)

To make our results comparable to those in Gomme (1993), we follow him by choosing \( \omega = 0.2281 \) and \( \sigma = 3.1922 \).\(^{13}\)

For the second modification to the basic model, we consider a CIA specification in which purchases of both consumption and investment goods must be made with currency, but in which cash has a differential productivity between consumption and investment purchases – we term this case CIA in everything. Formally,

\[ P_t(c_t + \eta i_t) \leq m_t + \tau_t, \]  

(32)

where \( 0 < \eta < 1 \) is a constant. When \( \eta = 0 \), (32) becomes the CIA in consumption. To illustrate, we choose \( \eta = 0.2 \) and 0.4 to compute the welfare costs.

As for the third modification, we follow Lucas (1994) and Goodfriend (1997) to specify the transactions technology as follows:

\[ P_t(c_t) \leq \psi(m_t + \tau_t)h_{ct}, \]  

(33)

and the instantaneous utility function is modified as

\[ U(c, h_f, h_c) = \ln c - B(h_f + h_c), \]  

(34)

where \( h_c \) and \( h_f \) are the fractions of time that the household devotes to shopping and working, respectively, and \( \psi > 0 \) a constant parameter. We use the actual U.S. inflation rate to form an estimate of \( \psi \). From 1954.I to 1991.III, the U.S. average annual CPI inflation rate is 4.11%. Using our basic CIA in consumption model, this implies a steady-state value of labor effort equal to 0.290. We set \( h_f \) equal to this steady-state value of labor effort. This value, along with the 4.11% inflation rate jointly produces \( \psi = 312 \) from the steady-state formulae.

Table 3 depicts the welfare costs of a 10% annual inflation rate for the above three model specifications, where the basic case of CIA in consumption is duplicated for ease of comparison. Several observations are worth making from these results.

First, with the CIA constraint binding, under the CES preferences, agents may enjoy a higher flexibility in substituting leisure for consumption than under the

\(^{13}\) We experiment with other values of these parameters and find the results insensitive to the choices of parameters.
indivisible labor case, and may therefore entail a lower welfare cost for a given inflation level. Indeed, we find in Table 3 that the cost is slightly lower under the CES utility ($\Delta c/c^A = 3.293\%$, and $\Delta c/Y^A = 2.749\%$, as compared to $\Delta c/c^A = 4.100\%$, and $\Delta c/Y^A = 3.422\%$ in the indivisible labor case), confirming this intuition.

Second, when $\eta > 0$, (32) places a more stringent constraint than the CIA in consumption only case (2) on economic agents’ choices, and should create a higher welfare cost. Furthermore, the larger the $\eta$, the higher the cost. The cost measures reported in Table 3 justify this claim.

Third, our shopping-time specification produces $\Delta c/c^A = 3.940\%$, and $\Delta c/Y^A = 3.289\%$, which are very close to those in the CIA in consumption case. It is interesting to compare these estimates with those in two recent studies using the same type of transactions constraint, namely Lucas (1994) and Goodfriend (1997). Lucas (1994) uses the basic framework proposed by McCallum and Goodfriend (1987), to estimate the welfare cost. He introduces a transactions constraint to relate household holdings of real balances and the fraction of time spent on transacting to the spending flow carried out by the household, and estimates the welfare cost of inflation as the time spent on economizing on cash use. Lucas finds that for a 10% inflation, this cost is about 1.3% of GDP, which is higher than his previous finding of 0.45% with a CIA in consumption constraint (Lucas, 1981). In contrast, the introduction of monopolistic competition and increasing returns in our model greatly enlarges the welfare cost estimate (2% higher than Lucas’s).

More recently, Goodfriend (1997) studies the welfare cost of inflation using the same transactions technology. He introduces monopolistic competition (and hence a non-unitary price-marginal cost markup) and endogenous labor supply into his model but assumes capital away, and shows that the welfare cost is proportional to the markup. Our estimate of the welfare cost confirms that of Goodfriend, although ours is higher. It should be pointed out that our model differs from Goodfriend’s in two important dimensions which may conceivably account for the difference in the welfare cost estimates. First, we model

<table>
<thead>
<tr>
<th>Model specification</th>
<th>CIA in consumption</th>
<th>CES utility</th>
<th>CIA in everything</th>
<th>Shopping time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta = 0.2$</td>
<td>$\eta = 0.4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\Delta c/c^A) \times 100$</td>
<td>4.100</td>
<td>3.293</td>
<td>4.605</td>
<td>5.109</td>
</tr>
<tr>
<td>$(\Delta c/Y^A) \times 100$</td>
<td>3.422</td>
<td>2.749</td>
<td>3.849</td>
<td>4.276</td>
</tr>
</tbody>
</table>
monopolistic competition within the framework of Benhabib and Farmer (1994) and our production technology is subject to strong increasing returns to scale; and second we allow capital accumulation in our economy. The latter invokes further effects of inflation through output on consumption.

Finally, notwithstanding the differences in model specifications, the welfare cost estimates for a 10% inflation under our sunspot economy are in the same order of magnitude and are large, thereby suggesting that our results are quite robust and general.

7. Concluding remarks

This paper has introduced new elements – monopolistic competition and increasing returns to scale – into a standard RBC model with a cash-in-advance constraint to re-estimate the welfare costs of inflation. Under such an alternative hypothesis, the model economy is capable of generating the observed aggregate fluctuations even when there are no shocks to the fundamentals. In particular, we demonstrate that this model matches the stylized U.S. business cycles facts as well as two more standard models, which forms the basis for our reexamination of the issue of interest. In such a paradigm, it is found that the welfare cost of inflation depends on the magnitudes of scale parameters as well as on the degree of competition among firms, and that the cost is considerably higher than that in standard monetary RBC models.

The modest or small estimated welfare costs of inflation reported in the literature are inconsistent with the fact that people attach to inflation significantly. If these small welfare results were true, the policy implication would be that inflation should have never become a major social problem. However, people have shown a strong revealed preference for low rates of inflation, by their willingness to incur large costs to achieve a lower rate. For example, in the United States in the 1970s, when the rate of inflation was significantly low by comparison with many historical episodes in other countries, often a substantial majority of respondents to the Gallop Poll, sometimes as large as 80%, named inflation as ‘the most important problem facing the country’ (see the analysis by Fischer and Huizinga, 1982). To unravel this, we have explored the effects of inflation in a monopolistically competitive economy and obtained larger welfare results than those documented in existing studies.

It should also be pointed out that in our model the indeterminacy of equilibria arises from the shocks to agents’ belief. This is consistent with the existing framework in monetary economics (e.g., see Farmer, 1995, Chapter 9). Moreover, our exercise enriches the literature, in that we have not only proposed a monetary model in which sunspots matter, but also demonstrated that this economy displays similar characteristics of business fluctuations to the actual U.S. economy.
Conceivably, there are two potential extensions. One approach is to endogenize each firm’s price markups over costs. Although we have experimented with varying markups in our simulations and welfare computations, there still lacks an endogenous mechanism. Since many economists believe that cyclical variations in markups are an important feature of business cycles (e.g., Rotemberg and Woodford, 1991), it would be interesting to explore the implications of endogenous markups in our context of monopolistic competition and increasing returns to scale. A second approach is to incorporate either costless credit (e.g., Cooley and Hansen, 1991) or costly credit (e.g., Gillman, 1993). These extensions should enable us to understand better the effects of inflation in alternative economies.

References


14 We are indebted to Dean Corbae for this suggestion.


