1D Problems

First recall the use of derivatives in simple one-dimensional (one variable) problems. The equations for a line, for example, and its slope (the derivative) are

\[ F = ax + b \]
\[ \frac{dF}{dx} = a \]  

(1)

The slope of the function \( F \) along slices parallel to the x-axis at a fixed value of \( y \) is called a \textit{partial derivative}, \( \frac{\partial F}{\partial x} \). Another partial derivative of \( F \) can be written for the slope of slices parallel to the y-axis, \( \frac{\partial F}{\partial y} \). The subscripts \( x \) or \( y \) simply serve as reminders that the one variable is treated as a constant when computing the derivative with respect to the other variable.

2D Example

Now imagine a simple two-dimensional (two variables) problem. The simple equation describing a plane can be used to illustrate the notion of partial derivatives.

\[ F = ax + by + c \]
\[ \left( \frac{\partial F}{\partial x} \right)_y = a \]
\[ \left( \frac{\partial F}{\partial y} \right)_x = b \]  

(2)

The total derivative of \( F \) is a tiny piece of the surface defined by the partial derivatives:

\[ dF = \left( \frac{\partial F}{\partial x} \right)_y \, dx + \left( \frac{\partial F}{\partial y} \right)_x \, dy \]  

(3)

It is straightforward to extend the two-dimensional case to problems with many dimensions.